VELS

INSTITUTE OF SCIENCE, TECHNOLOGY & ADVANCED STUDIES (VISTAS)
(Deemed to be University Estd. u/s 3 of the UGC Act, 1956)
PALLAVARAM - CHENNAI

DCMBA-21

Quantitative Techniques

MBA
ODL MODE
(Semester Pattern)

School of Management Studies and Commerce
Centre for Distance and Online Education
Vels Institute of Science, Technology and Advanced Studies (VISTAS)
Pallavaram, Chennai - 600117

Vels Institute of Science, Technology and Advanced Studies

Centre for Distance and Online Education

Master of Business Administration (MBA) ODL Mode

(Semester Pattern)

DCMBA-21: Quantitative Techniques

(4 Credits)

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Printed at:

FOREWORD

Dr.Ishari K Ganesh
Chancellor

Vels Institute of Science, Technology and Advanced Studies (VISTAS), Deemed-to-be University, was established in 2008 under section 3 of the Act of 1956 of the University Grants Commission (UGC), Government of India, New Delhi.

VISTAS has blossomed into a multi-disciplinary Institute offering more than 100 UG & PG Programmes, besides Doctoral Programmes, through 18 Schools and 46 Departments. All the Programmes have the approval of the relevant Statutory Regulating Authorities such as UGC, UGC-DEB, AICTE, PCI, BCI, NCTE and DGS.

Our University aims to provide innovative syllabi and industry-oriented courses, and hence, the revision of curricula is a continuous process. The revision is initiated based on the requirement and approved by the Board of Studies of the concerned Department/School. The courses are under Choice Based Credit Systems, which enables students to have adequate freedom to choose the subjects based on their interests.

I am pleased to inform you that VISTAS has been rendering its services to society to democratize the opportunities of higher education for those who are in need through Open and Distance Learning (ODL) mode.

VISTAS ODL Programmes offered have been approved by the University Grants Commission (UGC) – Distance Education Bureau (DEB), New Delhi.

The Curriculum and Syllabi have been approved by the Board of Studies, Academic Council, and the Executive Committee of the VISTAS, and they are designed to help provide employment opportunities to the students.

The MBA ODL Programme Study Materials have been prepared in the Self Instructional Mode (SIM) format as per the UGC-DEB (ODL & OL) Regulations 2020. It is highly helpful to the students, faculties and other professionals. It gives me immense pleasure to bring out the ODL programme with the noble aim of enriching learners' knowledge. I extend my congratulations and appreciation to the Programme Coordinator and the entire team for bringing up the ODL Programme in an elegant manner.

At this juncture, I am glad to announce that the syllabus of this ODL Programme has been made available on our website, **www.vistascdoe.in**, for the benefit of the student community and other knowledge seekers. I hope that this Self Learning Materials (SLM) will be a supplement to the academic community and everyone.

CHANCELLOR

FOREWORD

Dr.S.Sriman Narayanan
Vice-Chancellor

My Dear Students!

Open and Distance Learning (ODL) of VISTAS gives you the flexibility to acquire a University degree without the need to visit the campus often. VISTAS-CDOE involves the creation of an educational experience of qualitative value for the learner that is best suited to the needs outside the classroom. My wholehearted congratulations and delightful greetings to all those who have availed themselves of the wonderful leveraged opportunity of pursuing higher education through this Open and Distance Learning Programme.

Across the World, pursuing higher education through Open and Distance Learning Systems is on the rise. In India, distance education constitutes a considerable portion of the total enrollment in higher education, and innovative approaches and programmes are needed to improve it further, comparable to Western countries where close to 50% of students are enrolled in higher education through ODL systems.

Recent advancements in information and communications technologies, as well as digital teaching and e-learning, provide an opportunity for non-traditional learners who are at a disadvantage in the Conventional System due to age, occupation, and social background to upgrade their skills.

VISTAS has a noble intent to take higher education closer to the oppressed, underprivileged women and the rural folk to whom higher education has remained a dream for a long time.

I assure you all that the Vels Institute of Science, Technology and Advanced Studies would extend all possible support to every registered student of this Deemed-to-be University to pursue her/his education without any constraints. We will facilitate an excellent ambience for your pleasant learning and satisfy your learning needs through our professionally designed curriculum, providing Open Educational Resources, continuous mentoring and assessments by faculty members through interactive counselling sessions.

VISTAS, Deemed- to- be University, brings to reality the dreams of the great poet of modern times, Mahakavi Bharathi, who envisioned that all our citizens be offered education so that the globe grows and advances forever.

I hope that you achieve all your dreams, aspirations, and goals by associating yourself with our ODL System for never-ending continuous learning.

With warm regards,

VICE-CHANCELLOR

Course Introduction

This Course "**DCMBA-21-Quatitative Techniques"** mainly focuses on developing the Mathematical skill for the MBA student, especially in data analysis. There should be clear, and well developed strategies for improving the overall model for any business transaction. The entire course is developed to provide an insight knowledge of resource allocation, optimization. The 20 units in the five blocks comprise the overall course of Quantitative Techniques.

Block-1:Linear Programming has been divided in to four Units (Unit-1 to Unit-4). Unit-1 discuss about the basic essentials of Operation research, Unit-2 deals with Linear Programming Problems (LPP) and its Formulation, Unit-3 provides an introduction to Graphical Method and the Unit-4 describes about Simplex Method.

Block-2: **Transportation and Assignment Problems** has been divided in to four Units (Unit-5 to Unit-8). Unit-5 deals with Transportation Problems, Unit-6 explains with Initial Solution: North West Corner Rule and Least Cost Method, Unit-7 presents the Initial Solution: Vogel's Approximation Method and Unit-8 discuss with Balanced and Unbalanced Assignment Problems.

Block-3: **Network Models** has been split into four Units(Unit-9 to Unit-12).Unit- 9 discuss the Basic Terminologies, Unit-10 deals with Network Models, Unit- 11 describes about Critical Path Method (CPM) and Unit- 12 explains about Program Evaluation Review Technique (PERT).

Block-4:Game Theory has been split into four Units (Unit-13 to Unit-16). Unit-13 deals with Zero games and non-zero games, Unit- 14 explains about Pure and mixed strategy, Unit-15 describes about the concept of Maximin–Minimax Principle and Unit-16 explains about Dominance Property.

Block-5:Queuing and Simulation has been divided into four Units (Unit-17 to Unit-20). Unit- 17 deals with Queuing System, Unit-18 explains about Queuing models: Birth and Death Model, Unit-19 speaks bout simulation and Unit-20 discuss about Simulation Models.

DCMBA-21: Quantitative Techniques

Block-1: Introduction

Block-1: Linear Programming has been divided in to four Units. **Unit-1: Operations Research** explains about Introduction, Tools and Techniques, Operations Research Models and Applications.

Unit-2: Linear Programming Problems (LPP) and its Formulation describes about Introduction of LPP, Mathematical Formulation of Linear, Programming Problem, Steps for formulation of Linear Programming Problem and Solved Problem.

Unit-3: Graphical Method presents about the Introduction of Graphical Methods, Steps for solving the Linear Programming Problem using the graphical method and Solved Problem

Unit-4: Simplex Method deals with Introduction of Simplex Method, Steps for solving the Linear Programming Problem using the simplex method and Solved Problem.

In all the units of Block -1 **Linear Programming**, the Check your progress, Glossary, Answers to Check your progress and Suggested Reading has been provided and the Learners are expected to attempt all the Check your progress as part of study.

Unit-1

Operations Research

STRUCTURE

Overview

In this units we are going to discuss the tools, operations research models, applications of Operations Research. An overall view of the history of Operation research is given here.

Objectives

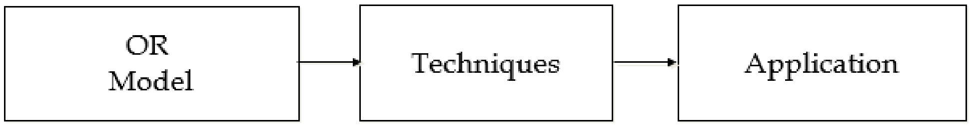
After studying this unit, the students will be able to

- understand the meaning, tools, model and
- the applications of Operations Research.

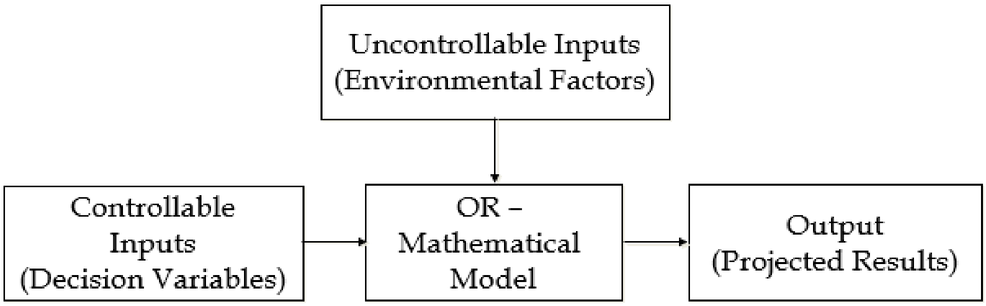
1.1. Introduction

Operations research is defined as the scientific process of transforming data into insights to making better decisions. It starts when mathematical and quantitative techniques are used to substantiate the decision being taken. The main activity of a manager is the decision making. In our daily life we make the decisions even without noticing them. The decisions are taken simply by common sense, judgment and expertise without using any mathematical or any other model in simple situations. But the decision we are concerned here with are complex and heavily responsible. Examples are public transportation network planning in a city having its own layout of factories, residential blocks or finding the appropriate product mix when there exists a large number of products with different profit contributions and production requirement etc. Operations research is decision-making tool that seeks for the best outcomes while keeping in mind the organization's overall goals and limits.

Operations Research Models



Model is an abstract representation of reality. The following factors can be used to classify models:



Abstraction (Degree)

I. Mathematical Models

The mathematical model is one which employs a set of mathematical symbols to represent the decision variables of the system. These variables are related together by means of a mathematical equation or a set of equations to describe the properties of the system. The solution of the problem is then obtained by applying well-developed mathematical techniques to the model.

ii. Analogue Models

The models in which one set of properties is used to represent another set of properties are called analogue models.

Goal:

i. Descriptive Models

A descriptive model simply describes some aspects of a situation based on observations, survey, questionnaire results or other available data. The result of an opinion poll represents a descriptive model.

ii. Predictive Models

Predictive model is one which predicts something based on some data. Predicting election results before the counting is completed.

Nature of Environment:

i. Deterministic Models

Deterministic models are a model which does not take uncertainty into account.

ii. Probabilistic Models

These types of models usually handle such situations in which the consequences or payoff of managerial actions cannot be predicted with certainty. However, it is possible to forecast a pattern of events, based on which managerial decisions can be made.

Time Horizon:

i. Static Models

These models do not consider the impact of changes that takes place during the planning horizon, i.e. they are independent of time. Also, in a static model only one decision is needed for the duration of a given time.

ii. Dynamic Models

In these models, time is considered as one of the important variables and admits the impact of changes generated by time. Also, in dynamic models, not only one but series of independent decisions is required during the planning horizon.

1.2. Tools and Techniques

Operations Research uses any suitable tools or techniques available. The common frequently used tools/techniques are mathematical procedures, cost analysis, electronic computation. However, operations researchers given special importance to the development and the use of techniques like linear programming, game theory, decision theory, queuing theory, inventory models and simulation. In addition to the above techniques, some other common tools are non-linear programming, integer programming, dynamic programming, sequencing theory, Markov process, network scheduling (PERT/CPM), symbolic Model, information theory, and value theory. There is many other Operations Research tools/techniques also exists. The brief explanations of some of the above techniques/tools are as follows:

- Linear Programming: This is a constrained optimization technique, which optimize some criterion within some constraints. In Linear programming the objective function (profit, loss or return on investment) and constraints are linear. There are different methods available to solve linear programming.
- Game Theory: This is used for making decisions under conflicting situations where there are one or more players/opponents. In this the motive of the players are dichotomized. The success of one player tends to be at the cost of other players and hence they are in conflict.
- Decision Theory: Decision theory is concerned with making decisions under conditions of complete certainty about the future out comes and under conditions such that we can make some probability about what will happen in future.
- Queuing Theory: This is used in situations where the queue is formed (for example customers waiting for service, air craft's waiting for landing, jobs waiting for processing in the computer system, etc.). The objective here is minimizing the cost of waiting without increasing the cost of servicing.
- Inventory Models: Inventory model make a decisions that minimize total inventory cost. This model successfully reduces the total cost of purchasing, carrying, and out of stock inventory.
- Network Scheduling: This technique is used extensively to plan, schedule, and monitor large projects (for example computer system installation, R&D design, construction, maintenance, etc.). The aim of this technique is minimize trouble spots (such as delays, interruption, production bottlenecks, etc.) by identifying the critical factors. The different activities and their relationships of the entire project are represented diagrammatically with the help of networks and arrows, which is used for identifying critical activities and path. There are two main types of technique in network scheduling, they are:
- Program Evaluation and Review Technique (PERT): It is used when activities time is not known accurately/only probabilistic estimate of time is available.
- Critical Path Method (CPM): It is used when activities time is not accurate.

Deterministic Models – assume all data are known with certainty

- Linear Programming
- Network Optimization
- Integer Programming
- Inventory models
- Replacement models

Stochastic Models – explicitly represent uncertain data (random variables or stochastic processes)

- Queuing
- Game Theory
- Inventory models
- Simulation
- Markov Chains

Techniques – Basic

- Linear Programming – Graphical, Simplex, Dual
- Network Optimization – Transportation, Assignment, CPM, PERT
- Inventory models – Deterministic
- Replacement models
- Queuing models
- Simulation – Monte-Carlo

Techniques – Advanced

- Linear Programming – Revised Simplex, Primal-Dual
- Network Optimization – Minimal Spanning Tree, Shortest Path Algorithms, Maximal Flow Problems or Algorithms, Minimum Cost Flow
- Game theory
- Inventory models – Stochastic
- Simulation – Discrete-Event
- Integer Programming
- Dynamic Programming

1.2. Applications

Today, almost all fields of business and government utilizing the benefits of Operations Research. There are voluminous of applications of Operations Research. The following are the abbreviated set of typical operations research applications to show how widely these techniques are used today:

Accounting

- Assigning audit teams effectively
- Credit policy analysis
- Cash flow planning Developing standard costs
- Establishing costs for by products
- Planning of delinquent account strategy
- Construction
- Project scheduling, monitoring and control
- Determination of proper work force
- Deployment of work force
- Allocation of resources to projects
- Facilities Planning
- Factory location and size decision
- Hospital planning
- International logistic system design
- Transportation loading and unloading
- Warehouse location decision

Finance

- Building cash management models
- Allocating capital among various alternatives
- Building financial planning models
- Investment analysis
- Portfolio analysis
- Dividend policy making

Marketing

- Advertising budget allocation
- Product introduction timing
- Deciding most effective packaging alternative

Organizational Behavior

- Personnel planning
- Recruitment of employees
- Skill balancing
- Training program scheduling
- Designing organizational structure more effectively

Research and Development

- R&D Projects control
- R&D Budget allocation
- Planning of Product introduction

Let us sum up

In this unit, the students have learned about the meaning, tools, model and the applications of Operations Research.

Check your Progress

1.Operations Research approach is?

a. multi-disciplinary
b. scientific
c. intuitive
d. collect essential data

2.Operation research approach is typically based on the use of ___________.

a. physical model
b. mathematical model
c. iconic model
d. descriptive model

3.Mathematical model of linear programming problem is important because ____________

a. it helps in converting the verbal description and numerical data into a mathematical expression
b. decision-makers prefer to work with formal models
c. it captures the relevant relationship among decision factors
d. it enables the use of algebraic technique

Glossary

Operations research: The scientific process of transforming data into insights to making better decisions.

Mathematical model: A set of mathematical symbols to represent the decision variables of the system.

Optimization: The selection of a best element, with regard to some criterion, from some set of available alternatives.

Answers to Check Your Progress

1.a

2.b

3.c

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

Unit-2

Linear Programming Problems (LPP) and its Formulation

STRUCTURE

Overview

In this section we are going to learn about the linear programming problem and its applications. Formulation of the LPP is also going to be learnt in this section.

Objectives

After studying this unit, the students are able to

- identify the characteristics of Linear Programming Problem and
- formulate a Linear Programming Problem.

2.1. Introduction

Linear Programming is a special and versatile technique which can be applied to a variety of management problems viz. Advertising, Distribution, Investment, Production, Refinery Operations, and Transportation analysis. The linear programming is useful not only in industry and business but also in non-profit sectors such as Education, Government, Hospital, and Libraries. The linear programming method is applicable in problems characterized by the presence of decision variables.

The objective function and the constraints can be expressed as **linear functions** of the **decision variables**. The decision variables represent quantities that are, in some sense, controllable inputs to the system being modeled. An objective function represents some principal objective criterion or goal that measures the effectiveness of the system such as maximizing profits or productivity, or minimizing cost or consumption. There is always some practical limitation on the availability of resources viz. man, material, machine, or time for the system. These constraints are expressed as linear equations involving the decision variables. Solving a linear programming problem means determining actual values of the decision variables that optimize the objective function subject to the limitation imposed by the constraints.

The main important feature of linear programming model is the presence of linearity in the problem. The use of linear programming model arises in a wide variety of applications. Some model may not be strictly linear, but can be made linear by applying appropriate mathematical transformations. Still some applications are not at all linear, but can be effectively approximated by linear models. The ease with which linear programming models can usually be solved makes an attractive means of dealing with otherwise intractable nonlinear models.

2.2. Mathematical Formulation of Linear Programming Problem (LPP)

It is defined as the process of considering various inequalities in a situation and estimating the best value that may be obtained under those circumstances. The basic components of the LP are as follows:

i. Decision Variables

The variables whose values determined the solution of a problem are called **decision variables** of the problem.

ii. Constraints

Subject to a set of simultaneous linear equations (or inequalities) known as constraints**.**

iii. Objective Functions

Linear programming problem deals with the optimization (maximization or minimization) of a function of decision variables known as objective function.

If $x_j (j = 1,2,3, \dots, n)$, the n decision variables and if the system is subject to m Constraints, then the general Mathematical model can be written in the form:

$(or) Minimize Z = (x_1, x_2, x_3, \dots, x_n)$

Subjectto$c_i(x_1, x_2, x_3, \dots, x_n) \leq or = or \geq b_i (i = 1,2,3, \dots, m)$called Structural constraints and $x_1, x_2, x_3, \dots, x_n \geq 0$ called the non-negative restrictions.

2.3. Steps for formulation of Linear Programming Problem

Step 1: Identify the unknown decision variables to be determined and assign symbols to them.

Step 2: Identify all the restrictions or constraints in the problem and express them as linear equations or inequalities of decision variables.

Step 3: Identify the aim (or) objective and represent it also as a linear function of decision variables.

Step 3: Express the complete formulation of LPP as a general mathematical model.

2.4. Solved Problem

Problem 1: A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.

Machine	Time per unit (minutes)			Machine capacity (Minutes/day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430

It is required to determine the number of units to be manufactured for each product daily. The profit per unit of product 1, 2 and 3 is Rs. 4/-, Rs.3/- and Rs.6/- respectively. Formulate the problem as a Linear Programming Problem (LPP).

Solution:

1. Decision Variables:

   $x_1 be the no. of product$ 1 $to be produced$
   $x_2 be the no. of product$ 2 $to be produced$
   $x_3 be the no. of product$ 3 $to be produced$

2. Objective Function

   $MaxZ = 4x_1 + 3x_2 + 6x_3$Constraints – Resources and Non-negativity

   Non-negativity

   $2x_1 + 3x_2 + 2x_3 \leq 440$
   $4x_1 + 0x_2 + 3x_3 \leq 470$
   $2x_1 + 5x_2 + 0x_3 \leq 430$
   $x_1, _2, x_3 \geq 0$

   Mathematical Model Subject to

   $MaxZ = 4x_1 + 3x_2 + 6x_3$ $2x_1 + 3x_2 + 2x_3 \leq 440$
   $4x_1 + 3x_3 \leq 470$
   $2x_1 + 5x_2 \leq 430$
   $x_1, _2, x_3 \geq 0$

Problem 2: A company produces three products that are processed on three different machines. It is required to determine the number of units to be manufactured for each product daily. The cost per unit of product 1, 2 and 3 is Rs. 45/-, Rs. 50/- and Rs. 65/- respectively. The time required to manufacture one unit of each of the two products and the daily capacity of the three machines are given in the table below.

Machine	Time per unit (minutes)			Machine capacity (Minutes/day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430

Solution:

1. Decision Variables:

2. Objective Function

3. $x_1 be the no. of product\ 1\ to be produced$

$x_2 be the no. of product\ 2\ to be produced$

$x_3 be the no. of product\ 3\ to be produced$

$Min Z = 45x_1 + 50x_2 + 65x_3$

4. Constraints – Resources and Non-negativity

$2x_1 + 3x_2 + 2x_3 \geq 440$

$4x_1 + 0x_2 + 3x_3 \geq 470$

$2x_1 + 5x_2 + 0x_3 \geq 430$

Non-negativity

Mathematical Model Subject to

Let us Sum Up

$x_1, _2, x_3 \geq 0$

$Min Z = 45x_1 + 50x_2 + 65x_3$ $2x_1 + 3x_2 + 2x_3 \geq 440$

$4x_1 + 3x_3 \geq 470$

$2x_1 + 5x_2 \geq 430$

$x_1, _2, x_3 \geq 0$

In this unit, the students have learned to identify the characteristics of Linear Programming Problem and formulate a Linear Programming Problem.

Check your Progress

1.Operation research approach is D

a. Multi-disciplinary
b. Artificial
c. Intuitive
d. All of the above

2.A constraint in an LP model restricts D

a. value of the objective function
b. value of the decision variable
c. use of the available resources
d. all of the above

3.A feasible solution of LPP A

a. Must satisfy all the constraints simultaneously
b. Need not satisfy all the constraints, only some of them
c. Must be a corner point of the feasible region
d. all of the above

Glossary

Linear Programming: A constrained optimization technique, which optimize some criterion within some constraints.

Decision Variables: The variables whose values determined the solution of a problem are called decision variables of the problem.

Constraints: A set of simultaneous linear equations (or inequalities) known as constraints.

Answers to let us sum up

1.d

2.d

3.a

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

Unit-3
Graphical Method

STRUCTURE

Overview

In this unit we are going to discuss about the graphical method through which we can formulate the LPP. Using this technique we can solve the given problems in an effective manner.

Objectives

After studying this unit, the students are able to

- solve a Linear Programming Problem using the graphical method.

3.1. Introduction

A graphical method that provides a pictorial description of the problems and their solutions, as well as the basic factors involved in solving general LPP involving any finite number of variables, may effectively solve LPP involving only two variables. The Linear Programming with two variables can be solved graphically. The graphical method of solving linear programming problem is of limited application in the business problems as the number of variables is substantially large. If the linear programming problem has larger number of variables, the suitable method for solving is Simplex Method. The simplex method is an iterative process, through which it reaches ultimately to the minimum or maximum value of the objective function.

3.2. Steps for solving the Linear Programming Problem using the graphical method

Step 1: Consider the inequality constraints as equalities. In the XOY plane, draw the straight lines that correspond to each equality and non-negativity restriction by putting $x_1 = 0$, we get $x_2$and $x_2 = 0$ we get x_1.

Step 2: Find the bounded region for the variables values, which is the regionbounded by the lines established in step 1.

Step 3: Find the point of intersection of the bounded lines by solving the equations of the corresponding lines, and substituting these points in the objective function to get the values of the objective function.

Step 4: For maximization problem, the maximum value of Z is the optimum value. For minimization problem, the minimum value of Z is the optimum value.

3.3. Solved Problem

Problem 1: Solve the following LPP using Graphical Method.

$\boldsymbol{MaxZ = 3x_1 + 2x_2}$

Subject to $\boldsymbol{-2x_1 + x_2 \leq 1}$

$\boldsymbol{x_1 \leq 2}$

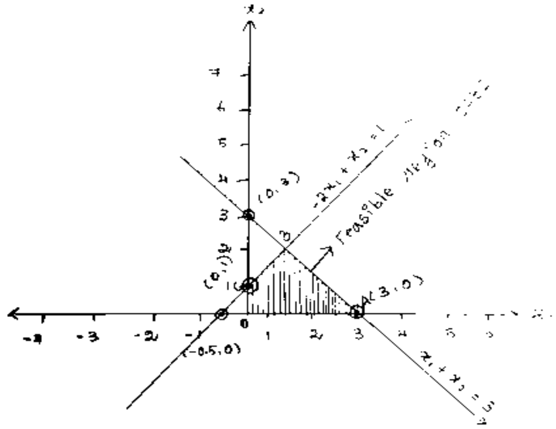
$\boldsymbol{x_1 + x_2 \leq 3}$**and** $\boldsymbol{x_1, x_2 \geq 0}$

Solution:

Consider the inequality constraints as equalities.

$\boldsymbol{-2x1 + x2 = 1}$	
Put $x_1 = 0$ $x_2 = 1$ $\Rightarrow (0,1)$	Put $x_2 = 0$ $x_1 = -0.5$ $\Rightarrow (-0.5,0)$

$\boldsymbol{x_1 + x_2 = 3}$	
Put $x_1 = 0$ $x_2 = 3$ $\Rightarrow (0,3)$	Put $x_2 = 0$ $x_1 = 3$ $\Rightarrow (3,0)$



The feasible region is also known as solution space of the LPP. Every point within this area satisfies all the constraints.

Vertex	Value of Z
O(0,0)	0
A(2,0)	**6**
B(2/3,7/3)	20/3
C(0,1)	2

Since the problem is of maximization type, the optimum solution is

$$\boldsymbol{MaxZ = 6,\ x_1 = 2,\ x_2 = 0}$$

Problem 2: Solve the following LPP using Graphical Method.

$$\boldsymbol{MinZ = 4x_1 + 2x_2}$$

Subject to $\boldsymbol{x_1 + 2x_2 \geq 2}$

$$\boldsymbol{3x_1 + x_2 \geq 3}$$

$$\boldsymbol{4x_1 + 3x_2 \geq 6}$$

and $\boldsymbol{x_1, x_2 \geq 0}$

Solution:

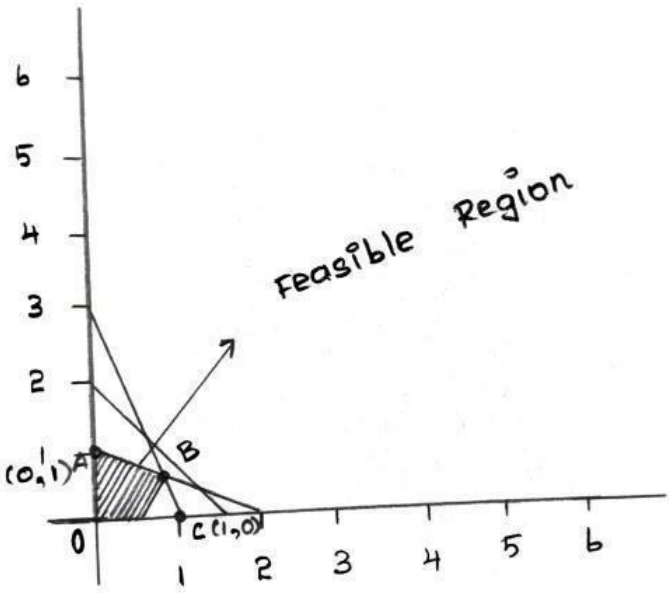
Consider the inequality constraints as equalities.

$\boldsymbol{x_1 + 2x_2 = 2}$	
Put $x_1 = 0$ $x_2 = 1$ ⇒ (0,1)	Put $x_2 = 0$ $x_1 = 2$ ⇒ (2,0)

$3x_1 + x_2 = 3$	
Put $x_1 = 0$ $x_2 = 3$ $\Rightarrow (0,3)$	Put $x_2 = 0$ $x_1 = 1$ $\Rightarrow (1,0)$

*

$4x_1 + 3x_2 = 6$	
Put $x_1 = 0$ $x_2 = 2$ $\Rightarrow (0,2)$	Put $x_2 = 0$ $x_1 = 1.5$ $\Rightarrow (1.5, 0)$



The feasible region is also known as solution space of the LPP. Every point within this area satisfies all the constraints.

Vertex	Value of Z
O(0,0)	0
A(0,1)	**2**
B(0.8,0.6)	4.4
C(1,0)	4

Since the problem is of minimization type, the optimum solution is

$$\boldsymbol{Min Z = 2,\ x_1 = 0,\ x_2 = 1}$$

Let us Sum Up

In this unit, the students have learned to solve a Linear Programming Problem using the graphical method.

Check your progress

1. The linear function of the variables which is to be maximize or minimize is called ____________
    a. Constraints
    b. Objective function
    c. Decision variable
    d. None of the above
2. A physical model is an example of __________
    a. An iconic model
    b. An analogue model
    c. A verbal model
    d. A mathematical model
3. A model is ___________
    a. An essence of reality
    b. An approximation
    c. An idealization
    d. All of the above

Glossary

Graphical Method: A pictorial description of the problems and their solutions, as well as the basic factors involved in solving general LPP involving any finite number of variables, may effectively solve LPP involving only two variables.

Answers to check your Progress

1.b

2.c

3. d

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

Unit-4

Simplex Method

STRUCTURE

Overview

In this chapter we are going to deal with the simplex method and the problems related to it. It is a simple method of LPP.

Objectives

After studying this unit, the students are able to

- solve a Linear Programming Problem using the simplex method.

4.1. Introduction

- Simplex Method is a mathematical procedure to solve the LPP involving more than two decision variables (x1, x2 ….. to any number of variables)
- It is the Standard technique in linear programming for solving an optimization problem, typically one involving a objective function and several constraints expressed as inequalities.
- It is developed to help solve large and real-world linear programming problems.

4.2. Steps for solving the Linear Programming Problem using the Simplex method

- It involves use of variables such as surplus, slack and artificial.

- It deals only with a small and unique set of feasible solutions (corner points).
- It is an iterative procedure carried systematically to determine the optimal solution from the set of feasible solutions.

Step 1: Check whether the objective function is to be maximized or minimized. If it is to be minimized, then convert it into a maximization type by Minimize Z = - Maximize (-Z)

Step 2: Check whether all RHS are positive. If any one of the RHS is negative, then multiply both sides of the constraint by -1.

Step 3: Convert inequality into equality constraints by introducing slack/surplus variables. **Step 4:** In the initial simplex table, take $x_1 = 0$ $and x_2 = 0$ so that $z = 0$ and the simplex table is as follows:

$(C_1 C_2 C_3 . . . . 0 0 0 . . . .)$

$C_B Y_B$	X_B	$x_1 x_2 x_3$ . . . . $s_1 s_2 s_3$ . . . .
$C_{B1}\ s_1$	b_1	$a_{11} a_{12} a_{13}$ . . . . 1 0 0 . . . .
$C_{B1}\ s_2$	b_2	$a_{21} a_{22} a_{23}$ . . . . 0 1 0 . . . .
$C_{B1}\ s_3$	b_3	$a_{31} a_{32} a_{33}$ . . . . 0 0 1 . . . .
:	:	: : : . . . .
:	:	. . . .
:		: : : . . . .
:		. . . .
$(Z_j - C_j)$	Z_0	$Z_1 - C_1 Z_2 - C_2$ … …... . . . . . .

Where C_j- row denotes the coefficients of the variables in the objective function

C_B- Column denotes the coefficients of the basic variables in the objective function

Y_B- Column denotes the basic variables

X_B- Column denotes the values of the basic variables.

Step 5: Compute the net evaluations $(Z_j - C_j)$

If all $(Z_j - C_j) \geq 0$ then the current basic feasible solution is optimal otherwise it is not optimal, go to next step.

Step 6: **To find the entering variable -** If more than one variable has the same most negative $(Z_j - C_j)$, any of these variables may be selected arbitrarily as the entering variables. The entering variable column is called as the key column or pivot column.

Step 7: **To find the leaving variable -** The ration between the solution column and the entering variable column by considering only positive denominators and is defined as

$\theta = Min \{{}^{XBi},{}^{a}ir ir > 0\}$

If $a_{ir} \leq 0$, then there is an unbounded solution to the given LPP.

The leaving variable row is called the **key row or pivot row or pivot equation**, and the element at the intersection of the pivot column and the pivot row is called the **pivot element.**

Step 8: Drop the leaving variable and introduce the entering variable along with its associated value under $C_B$column.

New pivot equation = Old pivot equation / pivot element New equation (all other rows including $(Z_j - C_j)$row)

= (Old equation) – {(Corresponding column coefficient) X (New pivot equation)}

Step 9: Go to step 5 and repeat the procedure until either an optimum solution is bounded or there is an indication of an unbounded solution.

Note 1: For maximization problem:

i. If all $(Z_j - C_j) \geq 0$, then the current basic feasible solution is optimal.

   If at least one $(Z_j - C_j) < 0$, then the current basic feasible solution is not optimal.

ii. The entering variable is the non-basic variable corresponding to the most negative value of $(Z_j - C_j)$.

Note 2: For minimization problem:

i. If all $(Z_j - C_j) \leq 0$, then the current basic feasible solution is optimal.

   If at least one $(Z_j - C_j) > 0$, then the current basic feasible solution is not optimal.

ii. The entering variable is the non-basic variable corresponding to the most positive value of $(Z_j - C_j)$.

4.3. Solved Problem

Problem 1: Solve the following LPP using Simplex Method Maximize $Z = 21x_1 + 15x_2$

Subject to the constraints

$-x_1 - 2x_2 \geq -6$

$4x_1 +\ \ \leq$ **12 and** $x_1, x_2 \geq 0$

Solution:

Step (1): In first inequality, RHS is negative, so multiply both sides of that constraint by -1 so as to make its RHS positive.

Step (2): By introduce the slack variables$s_{1,2}$, the problem in standard form becomes

Maximize $Z = 21x_1 + 15x_2 + 0s_1 + 0s_2$

Subject to

$x_1 + 2x_2 + s_1 = 6$ $4x_1 + 3x_2 + s_2 = 12$ and$x_1, x_2, s_1, s_2 \geq 0$

Step (3): Put$x_1 = 0$ and $x_2 = 0$ in equation 1 and 2, we get the initial basic feasible solution is $s_1 = 6$ and $s_2 = 12$

Step (4): Initial Iteration

The initial simplex table is given by

C_J			21	15	0	0	RatioθX_{Bi}= Min {, air
C_B	Y_B	X_B	$\boldsymbol{x_1}$	x_2	s_1	s_2	^{a}ir> 0}
0	s_1	6	**1**	2	1	0	^{6}= 61
0	$\boldsymbol{s_2}$	**12**	**4**4	**3**	**0**	**1**	**^{12}= 3 ←4**
Z_J		0	**0**	0	0	0	
$Z_J - C_J$		0	**- 21**t	- 15	0	0	

Since $Z_j -\ \ <0$, the current basic feasible solution is not optimal.

Since $Z_J - C_J$= -21 is most negative, the corresponding non-basic variable $x_1$enters into the basis and the leaving variable is the basic variable $s_2$which corresponds to the minimum ratioθ = 3. Here the pivot element is 4

Step (5): First Iteration

New Pivot equation = Old pivot equation ÷ Pivot element

= (12 4 3 0 1) ÷ (4) = (3 1 ¾0 ¼)

New $s_1$equation = (Old $s_1$equation) – {(Corresponding column coefficient)×(New Pivot equation) }

= (6   12   1   0) – { (1) × ((3   1   ¾   0 ¼)

= (3   05/41 -1/4)

C_J			21	15	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
0	s_1	3	0	$\frac{5}{4}$	1	$\frac{-1}{4}$
21	x_1	3	1	$\frac{3}{4}$	0	$\frac{1}{4}$
Z_J		63	21	$\frac{63}{4}$	0	$\frac{21}{4}$
$Z_J - C_J$		0	0	$\frac{3}{4}$	0	$\frac{21}{4}$

Since all Z_j −   ≥0 , then the current basic feasible solution is optimal.

∴The optimal solution is Max Z= 63, x_1= 3 and x_2= 0.

Example 2: Solve the following LPP using Simplex Method
Minimize $Z = 8x_1 - 2x_2$

Subject to the constraints

$-4x_1 + 2x_2 \leq 1$

$5x_1$ −   ≤ 3 andx_1, $x_2 \geq 0$

Solution:

Step (1): Since the given objective function is of minimization type, we shall convert it into a maximization type as follows:

Maximize (-Z) = MaximizeZ^*= $-8x_1 + 2x_2$

Step (2): By introduce the slack variables$s_{1,2}$, the problem in standard form becomes Maximize Z^* = $-8x_1 + 2x_2 + 0s_1 + 0s_2$

Subject to

$-4x_1 + 2x_2 + s_1 = 1$

5x1 − 4x2 + s2 = 3 andx_1, x_2 ,$_1$, $s_2 \geq 0$

Step (3): Putx_1 = 0 and x_2 = 0 in equation 1 and 2, we get the initial basic feasible solution is s_1 = 1 and s_2 = 3

Step (4): Initial Iteration

The initial simplex table is given by

C_J			-8	2	0	0	Ratio $X_{Bi}\theta = Min$ {, a_{ir} ^{a}ir> 0}
C_B	Y_B	X_B	x_1	$\boldsymbol{x_2}$	s_1	s_2	
0	$\boldsymbol{s_1}$	**1**	**-4**	**2** 2	**1**	**0**	**1= 0.5←2**
0	s_2	3	5	**-4**	0	1	–
Z_J	0	0	**0**	0	0	Z_J	
$Z_J - C_J$	0	8	**-2**t	0	0	$Z_J - C_J$	

Since $Z_j - C_j$<0, the current basic feasible solution is not optimal.

Since $Z_J - C_J$= -2 is most negative, the corresponding non-basic variable $x_2$enters into the basis and the leaving variable is the basic variable $s_1$which corresponds to the minimum ratioθ = 0.5. Here the pivot element is 2

Step (5): First Iteration

New Pivot equation = Old pivot equation ÷ Pivot element

= (1 -4 2 1 0) ÷ (2) = (1/2 -2 11/2 0)

New $s_2$equation = (Old $s_2$equation) – {(Corresponding column coefficient)× (New Pivot equation) }

= (3 5 -4 0 1) – { (-4) × (1/2 -2 11/2 0)}

= (5 -302 1)

			-8	2	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
2	x_2	1 ----- 2	-2	1	1 ------ 2	0
0	s_2	5	-3	0	2	1
Z_J		1	-4	2	1	0
$Z_J - C_J$		1	4	0	1	0

Since all $Z_j -$ ≥ 0, then the current basic feasible solution is optimal.

∴The optimal solution is

$2x_1 + x_2 + s_1 = 50$

$2x_1 + 5x_2 + s_2 = 100$

$2x_1 + 3x_2 + s_3 = 90$ and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Step (2): Put $x_1 = 0$ and $x_2 = 0$ in equation 1 , 2 and 3,

we get the initial basic feasible solution is $s_1 = 50$, $s_2 = 100$ and $s_3 = 90$

Step (3): Initial Iteration

The initial simplex table is given by

C_J			4	10	0	0	0	Ratio θX_{Bi}= Min {, a_{ir} a_{ir}> 0}
C_B	Y_B	X_B	x_1	$\boldsymbol{x_2}$	s_1	s_2	s_3	
0	s_1	50	2	**1**	1	0	0	50= 50 1
0	s_1	50	2	**1**	1	0	0	50= 50 1
0	$\boldsymbol{s_2}$	**100**	**2**	**5**5	**0**	**1**	**0**	**100= 20 ←5**
0	s_3	90	2	**3**	0	0	1	90= 30 3
Z_J		0	0	**0**	0	0	0	
$Z_J - C_J$		0	- 4	**-10**t	0	0	$Z_J - C_J$	

Since Z_j − <0 , the current basic feasible solution is not optimal.

Since $Z_J - C_J$= -10 is most negative, the corresponding non-basic variable x_2 enters into the basis and the leaving variable is the basic variable s_2 which corresponds to the minimum ratio θ = 20. Here the pivot element is 5

Step (5): First Iteration

New Pivot equation = Old pivot equation ÷ Pivot element

= (100 2 5 0 1 0) ÷ (5) = (20 2/5 1 01/5 0)

New s_1 equation = (Old s_1 equation) – {(Corresponding column coefficient)×(New Pivot equation) }

= (50   2 1 1   0   0) – { 1) × (20 2/5   1   01/5 0)}

= (308/5   01-1/50)

New $s_3$equation = (Old $s_3$equation) – {(Corresponding column coefficient)× (New Pivot equation) }

= (90   2 3 0   0   1) – { 3) × (20 2/5   1   01/5 0)}

= (304/5   00-3/51)

C_J			4	10	0	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	30	$\frac{8}{5}$	0	1	$\frac{-1}{5}$	0
10	x_2	20	$\frac{2}{5}$	1	0	$\frac{1}{5}$	0
0	s_3	30	$\frac{4}{5}$	0	0	$\frac{-3}{5}$	1
Z_J		200	4	10	0	2	0
$Z_J - C_J$		200	0	0	0	2	0

Since all $Z_j -$ ≥0 , then the current basic feasible solution is optimal.

∴The optimal solution is Maximize$Z$= 200 ,$_1$= 0 and x_2= 20.

Let us Sum up

In this unit, the students have learned to solve a Linear Programming Problem using the simplex method.

Check your progress

1. Simplex method was designed by ____________
   a. Dantzig
   b. A.Charnes
   c. Lemke
   d. Hungarian
2. Graphical method of linear programming is useful when the number of decision variable are ___________
   a. 2

b. 2

c. finite

d. infinite

3. While solving a linear programming problem infeasibility may be removed by __________

   a. adding another constraint

   b. adding another variable

   c. removing a constraint

   d. removing a variable

Glossary

Basic Variable: Variable of a basic feasible solution has n non-negative value.

Non-Basic Variable: Variable of a feasible solution has a value equal to zero.

Artificial Variable: A non-negative variable introduced to provide basic feasible solution and initiate the simplex procedures.

Slack Variable: A variable corresponding to a ≤ type constraint is a non-negative variable introduced to convert the inequalities into equations.

Surplus Variable: A variable corresponding to a ≥ type constraint is a non-negative variable introduced to convert the constraint into equations.

Answers to check your progress

1.a

2.a

3.c

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, "Introduction to Operation Research", McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008.

Block-2: Introduction

Block 2 - Transportation and Assignment has been divided in to four Units.

Unit-5: Transportation Problems deals with Introduction of Transportation problems, Types of Transportation Problem, Mathematical Formulation of a Transportation Problem and Methods for finding Initial Solution.

Unit-6: Initial Solution: North West Corner Rule and Least Cost Method explains about the concept of North West Corner Method, Solved Problem, Least Cost Method and Solved Problem.

Unit-7: Initial Solution: Vogel's Approximation Method describes about Vogel's Approximation Method (VAM) and Solved Problem

Unit-8 : Balanced and Unbalanced Assignment Problems describes about Balanced Assignment Problem, Hungarian Method, Solved Problem, Unbalanced Assignment Problem and Solved Problem.

In all the units of Block -2 **Transportation and Assignment**, the Check your progress, Glossary, Answers to check your progress and Suggested Reading has been provided and the Learners are expected to attempt all the Check your progress as part of study.

Unit-5

Transportation Problems

STRUCTURE

Overview

In this unit we are going to learn the introduction to transportation problem

Objectives

After studying this unit, the students will be able to understand the meaning,

- types and methods of transportation problem.

5.1. Introduction

Transportation problem is a special kind of Linear Programming Problem (LPP) in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized.

5.2. Types of transportation problem

Balanced: When both supplies and demands are equal then the problem is said to be a balanced transportation problem.

Unbalanced: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.

5.3.Mathematical Formulation of a Transportation Problem

		Destination				Supply(s_i)
		D1	D2	D3	D4	
Source	O1	C_{11}	C_{12}	C_{13}	C_{14}	S1
	O2	C_{21}	C_{22}	C_{23}	C_{24}	S2
	O3	C_{31}	C_{32}	C_{33}	C_{34}	S3
	O4	C_{41}	C_{42}	C_{43}	C_{44}	S4
Demand (d_j):		d_1	d_2	d_3	d_4	

In the above table **D1**, **D2**, **D3** and **D4** are the destinations where the products/goods are to be delivered from different sources **S1**, **S2**, **S3** and **S4**. **Si** is the supply from the source **Oi**. **dj**is the demand of the destination **Dj**. **Cij**is the cost when the product is delivered from source **Si** to destination **Dj**.

Assume that there are m sources and n destinations.

Let ajbe the supply(capacity) at source i, bjbe the demand at destination j,cijbe the unit transportation cost from source i to destination j and xijbe the number of units shifted from source i to destination j.

Then the transportation problem can be expressed mathematically as Minimize Z = $\sum^{m}_{i-1}$

Then the transportation problem can be expressed mathematically as

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a_i$, i=1,2,3,…m.

$\sum_{i=1}^{m} x_{ij} = b_j$, j=1,2,3,…n.

And $x_{ij} \geq 0$, for all i and j.

Note that the two sets of constraints will be consistent if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

necessary and sufficient condition for a transportation problem to have a feasible solution. Problems satisfying this condition are called balanced transportation problems.

$\sum_{i=1}^{m} \neq \sum \quad a_i \qquad \qquad {}_{j=1}^{n}$

If then the transportation problem is said to be unbalanced.

5.4. Methods for finding Initial Solution

The initial solution is a significant step to achieve the minimal total cost (optimal solution) of the transportation problem. This involves Initial solution to the given balanced Transportation Problems. There are several methods available to obtain an initial basic feasible solution of a transportation problem. For finding the initial basic feasible solution total supply must be equal to total demand. The following methods are used to find the initial solutions.

1. North West Corner Method.
2. Least Cost Method.
3. Vogel's Approximation Method (VAM).

Let us Sum Up

In this unit, the students have learned to understand the meaning, types and methods of transportation problem.

Check your Progress

1. An artificial variable leaves the basis means, there is no chance for the ________ variable to enter once again.
   a. slack
   b. surplus
   c. artificial
   d. dual
2. Service mechanism in a queuing system is characterized by ______________.
   a. customers behavior
   b. servers behavior
   c. customers in the system
   d. server in the system
3. The objective of network analysis is to ___________.
   a. minimize total project duration
   b. minimize total project cost

c. minimize production delays, interruption, and conflicts

d. maximize total project duration

Glossary

Transportation problems: This is a type of LPP which is focused at transporting the goods and services at a minimum cost of time.

Balanced transportation problems: Here total supply equals to total demand.

Unbalanced transportation problems: Here total supply does not equal to total demand. It may be less or greater.

Answers to check your progress

1.c

2.b

3.c

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, "Introduction to Operation Research", McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

Unit-6

Initial Solution: North West Corner Rule and Least Cost Method

STRUCTURE

Overview

This chapter deals with the north west corner method which is the easiest method of Transportation Problem. The second method least cost method are also been dealt in this unit. Method and steps to solve the problems has been given in this section.

Objectives

After studying this unit, the students will be able to

- solve the initial solutions using North West Corner Rule and Least Cost Method.

6.1. North West Corner Method

The method starts at the North West (upper left) corner cell of the table.

Step-1: Allocate as much as possible to the selected cell, and adjust the associated amounts of capacity (supply) and requirement (demand) by subtracting the allocated amount.

Step-2: Cross out the row (column) with zero supply or demand to indicate that no further assignments can be made in that row (column). If both the row and column become zero simultaneously, cross out one of them only, and leave a zero supply or demand in the uncrossed-out row (column).

Step -3: If exactly one row (column) is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out. Go to step-1.Solved Problem

6.2. Solved Problem

Find the minimum cost of shipping after obtaining the initial solution for the following transportation problem using the North West Corner Rule. A company has manufacturing plants (A, B and C) with daily production of 7, 12 and 11 units and the warehouses (W1, W2 and W3) with a daily demand of 10, 10 and 10 units respectively. The transport cost in rupees per unit from each manufacturing location to each market are given below.

Plant	**Warehouse**			
	W1	**W2**	**W3**	
A	**1**	**2**	**6**	**7**
B	**0**	**4**	**2**	**12**
C	**3**	**1**	**5**	**11**
	10	**10**	**10**	

Step 1: Check for Balanced Transportation Problem (BTP)

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

7+12+11 = 10+10+10

30 = 30

Step 2: Solve the BTP by North West Corner Rule

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

North West Corner

$\boxed{1}$ 7	2	6	~~7~~ 0
0	4	2	12
3	1	5	11
~~10~~ 3	10	10	

$\boxed{0}$ 3	4	2	~~12~~ 9
3	1	5	11
~~3~~ 0	10	10	

$\boxed{4}$ 9	2	~~9~~ 0
1	5	11
~~10~~ 1	10	

1 1	5 10	~~11~~ 0
~~1~~ 0	~~10~~ 0	

1 **7**	**2**	**6**
0 **3**	**4** **9**	**2**
3	**1** **1**	**5** **10**

Step 3: Check for Non-degeneracy: N = m + n – 1

5 = 3 + 3 – 1

5 = 5

Step 4:

Calculate

the Total

Cost (TC)

TC = 1 x 7

+ 0 x 3 + 4

x 9 + 1 x 1

+ 5 x 10

= 94

6.3. Least Cost Method

The least cost method is also known as matrix minimum method in the sense we look for the row and the column corresponding to which Cijis minimum. This method finds a better initial basic feasible solution by concentrating on the cheapest routes. Instead of starting the allocation with the North West cell as in the North West Corner Method, we start by allocating as much as possible to the cell with the smallest unit cost. If there are two or more minimum costs then we should select the row and the column corresponding to the lower numbered row. If they appear in the same row we should select the lower numbered column. We then cross out the satisfied row or column, and adjust the amounts of capacity and requirement accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out. Next, we look for the uncrossed-out cell with the smallest unit cost and repeat the process until we are left at the end with exactly one uncrossed-out row or column.

6.4. Solved Problem

Find the minimum cost of shipping after obtaining the initial solution for the following transportation problem using the Least Cost method. A company has manufacturing plants (A, B and C) with daily production of 7, 12 and 11 units and the warehouses (W1, W2 and W3) with a daily demand of 10, 10 and 10 units respectively.

The transport cost in rupees per unit from each manufacturing location to each market are given below.

Plant	Warehouse			
	W1	**W2**	**W3**	
A	**1**	**2**	**6**	**7**
B	**0**	**4**	**2**	**12**
C	**3**	**1**	**5**	**11**
	10	**10**	**10**	

Step 1: Check for Balanced Transportation Problem (BTP)

$\boxed{1}$ 7	2	6	~~7~~ 0
0	4	2	12
3	1	5	11
~~10~~ 3	10	10	

7+12+11 = 10+10+10

30 = 30

Step 2: Solve the BTP by Least Cost Method

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

1	2	6	7
$\boxed{0}$ 10	4	2	2
3	1	5	11
0	10	10	

Least Cost

2	6	7
4	2	2
$\boxed{1}$ 10	5	1
0	10	

6 7	0
2 2	0
5 1	0
0	

1	2	6 7	7
0 10	4	2 2	12
3	1 10	5 1	11
10	10	10	

Step 3: Check for Non-degeneracy: N = m + n – 1

5 = 3 + 3 – 1

5 = 5

Step 4: Calculate the Total Cost (TC)

TC = 6 x 7 + 0 x 10 + 2 x 2 + 1 x 10 + 5 x 1 = 61

Let us Sum Up

In this unit, the students have learned to solve the initial solutions using North West Corner Rule and Least Cost Method.

Check Your Progress

(1).The transportation model is a special class of linear programming models.
True / False

(2).A transportation problem requires exactly as many origins as destinations.
True / False

(3).Neither the north west-corner rule nor the intuitive method considers shipping cost in making initial allocations.

True / False

Glossary

Feasible Solution: A feasible solution is said to be basic if the number of positive allocations equals m+n-1; that is one less than the number of rows and columns in a transportation problem.

Basic feasible solution: If the number of affirmative allocations equals m+n-1, the solution is considered to be Basic feasible solution

Answers to check your progress

1.True

2.False

3. False

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

Unit-7

Initial Solution: Vogel's Approximation Method

STRUCTURE

Overview:

In this unit we are going to deal with most important type of transportation problem which gives most opt solution for any problem.

Objectives

After studying this unit, the students will be able to

- solve the initial solutions using Vogel's Approximation Method (VAM).

7.1. Vogel's Approximation Method (VAM)

Vogel's Approximation Method (VAM) is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step-1: For each row (column) with strictly positive capacity (requirement), determine a **penalty** by subtracting the **smallest** unit cost element in the row (column) from the next **smallest** unit cost element in the same row (column).

Step-2: Identify the row or column with the **largest penalty** among all the rows and columns. If the penalties corresponding to two or more rows or columns are equal, we select the topmost row and the extreme left column.

Step-3: We select Xijas a basic variable if Cijis the **minimum cost** in the row or column with **largest penalty**. We choose the numerical value of Xijas high as possible subject to the row and the column constraints. Depending upon whether ai or bjis the smaller of the two i^{th} row or j^{th} column is crossed out.

Step 4: The Step 2 is now performed on the uncrossed-out rows and columns until all the basic variables have been satisfied

7.2. Solved Problem

Find the minimum cost of shipping after obtaining the initial solution for the following transportation problem using the Vogel's Approximation Method (VAM). A company has manufacturing plants (A, B and C) with daily production of 7, 12 and 11 units and the warehouses (W1, W2 and W3) with a daily demand of 10, 10 and 10 units respectively. The transport cost in rupees per unit from each manufacturing location to each market are given below.

Plant	Warehouse			
	W1	W2	W3	
A	1	2	6	7
B	0	4	2	12
C	3	1	5	11
	10	10	10	

Step 1: Check for Balanced Transportation Problem (BTP)

$\boxed{1}$ 7	2	6	~~7~~ 0
0	4	2	12
3	1	5	11
~~10~~ 3	10	10	

7+12+11 = 10+10+10

30 = 30

Step 2: Solve the BTP by Vogel's Approximation Method

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

			Penalty
1	2	7	2 − 1 = 1
$\boxed{0}$ 2	4	~~2~~ 0	4 − 0 = 4 Highest
3	1	11	3 − 1 = 2
~~10~~ 8	10		
1 − 0 = 1	2 − 1 = 1		

				Penalty
1	2	6	7	2 − 1 = 1
0	4	$\boxed{2}$ 10	~~12~~ 2	2 − 0 = 2
3	1	5	11	3 − 1 = 2
10	10	~~10~~ 0		
1 − 0 = 1	2 − 1 = 1	5 − 2 = 3 Highest		

Least Cost

1 7	~~7~~ 0
3 1	~~1~~ 0
~~8~~ 0	

1 7	2	6	7
0 2	4	2 10	12
3 1	1 10	5	11
10	10	10	

			Penalty
1	2	7	2 – 1 = 1
3	$\boxed{1}$ 10	~~11~~ 1	3 – 1 = 2 Highest
8	~~10~~ 0		
3 – 1 = 2 Highest	2 – 1 = 1		

Step 3: Check for Non-degeneracy: N = m + n – 1

5 = 3 + 3 – 1

5 = 5

Step 4: Calculate the Total Cost (TC)

TC = 1 x 7 + 0 x 2 + 3 x 1 + 1 x 10 + 2 x 10

= 40

Let us Sum Up

In this unit, the students have learned to solve the initial solutions using Vogel’s Approximation Method (VAM).

Check your Progress

(1).A feasible solution in transportation models is one in which all of the supply and demand constraints are satisfied.

True / False

(2).A transportation problem with a total supply of 500 and a total demand of 400 will have an optimal solution that leaves 100 units of supply unused.

True / False

(3).Which of the following is NOT needed in order to use the transportation model?.

a. the source points and their capacity.

b. the fixed costs of source points.

c. the cost of shipping one unit from each source to each destination.

d. the destination points and their demand.

e. All of these are needed.

Glossary

Degeneracy: Degeneracy occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus/

Dummy source: A dummy source or destination is added to balance transportation problem where demand is not equal to supply.

Answers to check your progress

1. True 2. True 3. b

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.

2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

3. Gupta. P. K, Man Mohan, Kanti Swarup: “Operations Research”, Sultan Chand, 2008.

Unit-8

Balanced and Unbalanced Assignment Problems

STRUCTURE

Overview

In this unit we are going to deal with balanced and unbalanced assignment problem where assignments of job is done to machine or person according to the need. Optimum cost is obtained after allocating.

Objectives

After studying this unit, the students will be able to

- solve the Assignment Problems using Hungarian Method.

8.1. The Balanced Assignment Problem

Given n facilities, n jobs and the effectiveness of each facility to each job, here the problem is to assign each facility to one and only one job so that the measure of effectiveness if optimized. Here the optimization means Maximized or Minimized. There are many management problems has an assignment problem structure. For example, the head of the department may have 6 people available for assignment and 6 jobs to fill. Here the head may like to know which job should be assigned to which person so that all tasks can be accomplished in the shortest time possible. Another example a container company may have an empty container in each of the location 1, 2,3,4,5 and requires an empty container in each of the locations 6,7,8,9,10. It would like to ascertain the assignments of containers to various locations so as to minimize the total distance. The third example here is, a marketing set up by making an estimate of sales performance for different salesmen as well as for different cities one could assign a particular sales man to a particular city with a view to maximize the overall sales.

Note that with n facilities and n jobs there are n! possible assignments. The simplest way of finding an optimum assignment is to write all the n! possible arrangements, evaluate their total cost and select the assignment with minimum cost. Bust this method leads to a calculation problem of formidable size even when the value of n is moderate. For n=10 the possible number of arrangements is 3268800.

If in an assignment problem some cost elements cij are negative, we may have to convert them into an equivalent assignment problem where all the cost elements are non-negative by adding a suitable large constant to the cost elements of the relevant row or column, and then we look for a feasible solution which has zero assignment cost after adding suitable constants to the cost elements of the various rows and columns. Since it has been assumed that all the cost elements are non-negative, this assignment must be optimum. On the basis of this principle a computational technique known as Hungarian Method is developed. The Hungarian Method is discussed as follows.

8.2. Hungarian Method

First check whether the number of rows is equal to the numbers of columns, if it is so, the assignment problem is said to be balanced.

Step :1 Choose the least element in each row and subtract it from all the elements of that row.

Step :2 Choose the least element in each column and subtract it from all the elements of that column. Step 2 has to be performed from the table obtained in step 1.

Step:3 Now determine an assignment as follows:

- For each row or column with a single zero element cells that has not be assigned or eliminated, box that zero element as an assigned cell.
- For every zero that becomes assigned, cross out all other zeros in the same row and for column.

- If for arrow and for a column there are two or more zero and one can't be chosen by inspection, choose the assigned zero cell arbitrarily.
- The above procedures may be repeated until every zero-element cell is either reassigned (boxed) or crossed out.

Step 4: If each row and each column contain exactly one assignment, then the solution is optimal.

Step 5: An optimum assignment is found, if the number of assigned cells is equal to the number of rows (and columns). In case we had chosen a zero cell arbitrarily, there may be an alternate optimum. If no optimum solution is found i.e. some rows or columns without an assignment then go to **Step6**.

Step 6: Draw a set of lines equal to the number of assignments which has been made in **Step 4**, covering all the zeros in the following manner.

- Mark check (√) to those rows where no assignment has been made.
- Examine the checked (√) rows. If any zero- element cell occurs in those rows, check (√) the respective columns that contains those zeros.
- Examine the checked (√) columns. If any assigned zero element occurs in those columns, check (√) the respective rows that contain those assigned zeros.
- The process may be repeated until now more rows or column can be checked.
- Draw lines through all unchecked rows and through all checked columns

Step7: Examine those elements that are not covered by a line. Choose the smallest of these elements and subtract this smallest from all the elements that do not have a line through them. Add this smallest element to every element that lies at the intersection of two lines. Then the resulting matrix is a new revised cost table.

8.3. Solved Problem

Solve the following assignment problem. Cell values represent cost of assigning job A, B, C and D to the machines I, II, III and IV.

		machines			
		I	II	III	IV
jobs	A	10	12	19	11
	B	5	10	7	8
	C	12	14	13	11
	D	8	15	11	9

Solution:

Here the number of rows and columns are equal.

∴The given assignment problem is balanced. Now let us find the solution.

Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

	I	II	III	IV
A	0	2	9	1
B	0	5	2	3
C	1	3	2	0
D	0	7	3	1

Look for at least one zero in each row and each column. Otherwise go to step 2.

Step 2: Select the smallest element in each column and subtract this from all the elements in its column.

	I	II	III	IV
A	0	0	7	1
B	0	3	0	3
C	1	1	0	0
D	0	5	1	1

Since each row and column contains at least one zero, assignments can be made.

Step 3 (Assignment):

Examine the rows with exactly one zero. First three rows contain more than one zero. Go to row D. There is exactly one zero. Mark that zero by □(i.e.) job D is assigned to machine I. Mark other zeros in its column by **×.**

	I	II	III	IV
A	$\cancel{0}$	0	7	1
B	$\cancel{0}$	3	0	3
C	1	1	0	0
D	$\boxed{0}$	5	1	2

Step 4: Now examine the columns with exactly one zero. Already there is an assignment in column I. Go to the column II. There is exactly one zero. Mark that zero by □. Mark other zeros in its row by **×**.

	I	II	III	IV
A	$\cancel{0}$	$\boxed{0}$	7	1
B	$\cancel{0}$	3	0	3
C	1	1	0	0
D	$\boxed{0}$	5	1	2

Column III contains more than one zero. Therefore, proceed to Column IV, there is exactly one zero. Mark that zero by □. Mark other zeros in its row by **×.**

	I	II	III	IV
A	$\cancel{0}$	$\boxed{0}$	7	1
B	$\cancel{0}$	3	0	3
C	1	1	$\cancel{0}$	$\boxed{0}$
D	$\boxed{0}$	5	1	2

Step 5: Again, examine the rows. Row B contains exactly one zero. Mark that zero by □.

	I	II	III	IV
A	$\cancel{0}$	$\boxed{0}$	7	1
B	$\cancel{0}$	3	$\boxed{0}$	3
C	1	1	$\cancel{0}$	$\boxed{0}$
D	$\boxed{0}$	5	1	2

Thus all the four assignments have been made. The optimal assignment schedule and total cost is

Job	Machine	cost
A	II	12
B	III	7
C	IV	11
D	I	8
Total cost		38

The optimal assignment (minimum) cost = ₹ 38

Example 2

Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows. Determine the optimum assignment schedule.

		Job				
		1	2	3	4	5
Person	*A*	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Solution:

Here the number of rows and columns are equal.

∴The given assignment problem is balanced. Now let us find the solution.

Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

The cost matrix of the given assignment problem is

		Job				
		1	2	3	4	5
Person	*A*	7	3	1	5	0
	B	0	9	5	5	4
	C	1	6	7	0	4
	D	4	3	1	0	3
	E	4	0	3	4	0

Column 3 contains no zero. Go to Step 2.

Step 2: Select the smallest element in each column and subtract this from all the elements in its column.

$$
\begin{array}{cc|ccccc|}
 & & \multicolumn{5}{c}{\text{Job}} \\
 & & 1 & 2 & 3 & 4 & 5 \\
\hline
 & A & 7 & 3 & 0 & 5 & 0 \\
 & B & 0 & 9 & 4 & 5 & 4 \\
\text{Person} & C & 1 & 6 & 6 & 0 & 4 \\
 & D & 4 & 3 & 0 & 0 & 3 \\
 & E & 4 & 0 & 2 & 4 & 0 \\
\hline
\end{array}
$$

Since each row and column contains at least one zero, assignments can be made.

Step 3 (Assignment):

Examine the rows with exactly one zero. Row B contains exactly one zero. Mark that zero by □.Person B is assigned to Job 1. Mark other zeros in its column by **×.**

$$
\begin{array}{cc|ccccc|}
 & & \multicolumn{5}{c}{\text{Job}} \\
 & & 1 & 2 & 3 & 4 & 5 \\
\hline
 & A & 7 & 3 & 0 & 5 & 0 \\
\text{Person} & B & \boxed{0} & 9 & 4 & 5 & 4 \\
 & C & 1 & 6 & 6 & 0 & 4 \\
 & D & 4 & 3 & 0 & 0 & 3 \\
 & E & 4 & 0 & 2 & 4 & 0 \\
\hline
\end{array}
$$

Now, Row C contains exactly one zero. Mark that zero by □. Mark other zeros in its column by ×.

$$\begin{array}{cc|ccccc|}
 & & \multicolumn{5}{c}{\text{Job}} \\
 & & 1 & 2 & 3 & 4 & 5 \\ \hline
 & A & 7 & 3 & 0 & 5 & 0 \\
 & B & \boxed{0} & 9 & 4 & 5 & 4 \\
\text{Person} & C & 1 & 6 & 6 & \boxed{0} & 4 \\
 & D & 4 & 3 & 0 & \cancel{0} & 3 \\
 & E & 4 & 0 & 2 & 4 & 0 \\ \hline
\end{array}$$

Now, Row D contains exactly one zero. Mark that zero by □. Mark other zeros in its column by **×.**

$$\begin{array}{cc|ccccc|}
 & & \multicolumn{5}{c}{\text{Job}} \\
 & & 1 & 2 & 3 & 4 & 5 \\ \hline
 & A & 7 & 3 & \cancel{0} & 5 & 0 \\
 & B & \boxed{0} & 9 & 4 & 5 & 4 \\
\text{Person} & C & 1 & 6 & 6 & \boxed{0} & 4 \\
 & D & 4 & 3 & \boxed{0} & \cancel{0} & 3 \\
 & E & 4 & 0 & 2 & 4 & 0 \\ \hline
\end{array}$$

Row E contains more than one zero, now proceed column wise. In column 1, there is an assignment. Go to column 2. There is exactly one zero. Mark that zero by □. Mark other zeros in its row by **×.**

$$\begin{array}{cc|ccccc|}
 & & \multicolumn{5}{c}{\text{Job}} \\
 & & 1 & 2 & 3 & 4 & 5 \\ \hline
 & A & 7 & 3 & \cancel{0} & 5 & 0 \\
 & B & \boxed{0} & 9 & 4 & 5 & 4 \\
\text{Person} & C & 1 & 6 & 6 & \boxed{0} & 4 \\
 & D & 4 & 3 & \boxed{0} & \cancel{0} & 3 \\
 & E & 4 & \boxed{0} & 2 & 4 & \cancel{0} \\ \hline
\end{array}$$

There is an assignment in Column 3 and column 4. Go to Column 5. There is exactly one zero. Mark that zero by □. Mark other zeros in its row by ×.

$$
\text{Person}\quad
\begin{array}{c|ccccc|}
\multicolumn{6}{c}{\text{Job}} \\
 & 1 & 2 & 3 & 4 & 5 \\
\hline
A & 7 & 3 & \cancel{0} & 5 & \boxed{0} \\
B & \boxed{0} & 9 & 4 & 5 & 4 \\
C & 1 & 6 & 6 & \boxed{0} & 4 \\
D & 4 & 3 & \boxed{0} & \cancel{0} & 3 \\
E & 4 & \boxed{0} & 2 & 4 & \cancel{0} \\
\hline
\end{array}
$$

Thus all the five assignments have been made. The Optimal assignment schedule and total cost is

Person	Job	cost
A	5	1
B	1	0
C	4	2
D	3	1
E	2	5
Total cost		9

The optimal assignment (minimum) cost = 9

8.4. Unbalanced Assignment Problem

Suppose if the number of persons is different from the number of jobs, then the assignment problem is called as **unbalanced**.

If the number of jobs is less than the number of persons, some of them can't be assigned any job. So that we have to introduce on or more dummy jobs of zero duration to make the unbalanced assignment problem into balanced assignment problem. This balanced assignment problem can be solved by using the Hungarian Method as discussed in the previous section. The persons to whom the dummy jobs are assigned left out of assignment.

Similarly, if the number of persons is less than number of jobs then we have introduced one or more dummy persons with zero duration to modify the unbalanced into balanced and then the problem is solved using the Hungarian Method. Here the jobs assigned to the dummy persons are left out.

8.5. Solved Problem

Example 3

Solve the following assignment problem.

		Men 1	2	3
Task	P	9	26	15
	Q	13	27	6
	R	35	20	15
	S	18	30	20

Solution:

Since the number of columns is less than the number of rows, given assignment problem is unbalanced one. To balance it, introduce a dummy column with all the entries zero. The revised assignment problem is here only 3 tasks can be assigned to 3 men.

		Men 1	2	3	d
Task	P	9	26	15	0
	Q	13	27	6	0
	R	35	20	15	0
	S	18	30	20	0

Step 1: is not necessary, since each row contains zero entry. Go to

Step 2 :

		Men 1	2	3	d
Task	P	0	6	9	0
	Q	4	7	0	0
	R	26	0	9	0
	S	9	10	14	0

Step 3 (Assignment):

Since each row and each column contains exactly one assignment, all the three men have been assigned a task. But task S is not assigned to any Man. The optimal assignment schedule and total cost is

Task	Men	cost
P	1	9
Q	3	6
R	2	20
S	*d*	0
Total cost		35

$$
\begin{array}{cc}
 & \text{Men} \\
 & \begin{array}{cccc} 1 & 2 & 3 & d \end{array} \\
\text{Task} \begin{array}{c} P \\ Q \\ R \\ S \end{array} & \left|\begin{array}{cccc} \boxed{0} & 6 & 9 & \cancel{0} \\ 4 & 7 & \boxed{0} & \cancel{0} \\ 26 & \boxed{0} & 9 & \cancel{0} \\ 9 & 10 & 14 & \boxed{0} \end{array}\right|
\end{array}
$$

The optimal assignment (minimum) cost = ₹ 35

Let us Sum Up

In this unit, the students have learned to solve the Assignment Problems using Hungarian Method.

Check your Progress

1.Which of the following is NOT information needed for a transportation problem?

a. the list of sources and the capacity at each
b. the set of destinations and the demand at each
c. the set of origins and the demand at each origin
d. the cost of shipping one unit from each origin to each destination
e. None of the above is needed

2.The purpose of the transportation approach for location analysis is to minimize which of the following?

a. the number of shipments
b. total fixed costs
c. total variable costs
d. total costs
e. total shipping costs

3.The transportation method is a special case of the family of problems known as what?

a. linear programming problems
b. statistical problems
c. decision tree problems
d. regression problems
e. simulation problems

Glossary

Cost Table: The completion time or cost corresponding to every assignment is written down in a table form if referred as a cost table.

Hungarian Method: A technique of solving assignment problems.

Assignment Problem: A special kind of linear programming problem where the objective is to minimize the assignment cost or time.

Balanced Assignment Problem: An assignment problem where the number of persons equal to the number of jobs.

Unbalanced Assignment

Problem: An assignment problem where the number of jobs is not equal to the number of persons.

Infeasible Assignment

Problem: An assignment problem where a particular person is unable to perform a particular job or certain job cannot be done by certain machines.

Answers to check your progress

1.c

2. e

3.a

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.

2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

3. Gupta. P. K, Man Mohan, Kanti Swarup: “Operations Research”, Sultan Chand, 2008.

Block-3: Introduction

Block-3: Network Models has been divided in to four Units. **Unit-9: Basic Terminologies** explains about the Basic Terminologies, Project Network and Solved Problem.

Unit-10: Network Models deals with Introduction, Network Models and Techniques of Network Models.

Unit-11: Critical Path method (CPM) describes about Critical Path Method (CPM) and Solved Problems.

Unit-12: Program Evaluation Review Technique (PERT) describes about the **Programme** Evaluation Review Technique (PERT) and Solved Problems

In all the units of Block -3 **Network Models**, the Check your progress, Glossary, Answers to Check your progress and Suggested Reading has been provided and the Learners are expected to attempt all the Check your progress as part of study.

Unit-9

Basic Terminologies

STRUCTURE

Overview

Objectives

9.1. Basic Terminologies

9.2. Project Network

9.3. Solved Problem

Let us Sum Up

Check your Progress

Glossary

Answers to check your progress

Suggestions

Overview

In this unit we are going to deal with the networking models to know the accurate path for any problem. Optimal cost is being obtained based on the network.

Objectives

After studying this unit, the students will be able to

- understand basic terminologies and Project Network of Network models.

9.1. Basic Terminologies Activity

An activity means a work. A project consists of several activities. An activity takes time. It is represented by an arrow in a diagram of the network. For example, an activity in house construction can be flooring. This is represented as follows:

Flooring $\longrightarrow$

Construction of a house involves various activities. Flooring is an activity in this project. We can say that a project is completed only when all the activities in the project are completed.

Event

It is the beginning or the end of an activity. Events are represented by circles in a project network diagram. The events in a network are called the **nodes**.

Example:

Starting a punching machine is an activity. Stopping the punching machine is another activity.

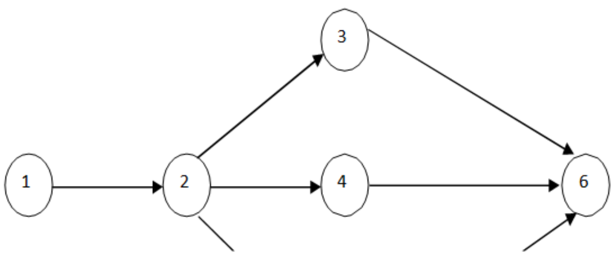
1. Predecessor Event

The event just before another event is called the predecessor event.

2. Successor Event

The event just following another event is called the successor event.

Example: Consider the following.



In this diagram, event 1 is predecessor for the event 2. Event 2 is successor to event 1.

Event 2 is predecessor for the events3,4 and 5. Event 4 is predecessor for the event 6.

Event 6 is successor to events 3,4 and 5.

Dummy Activity

A dummy activity is an activity which does not consume any time. Some times, it may be necessary to introduce a dummy activity in order to provide connectivity to a network or for the preservation of the logical sequence of the nodes and edges.

9.2. Project Network

A network consists of several destinations or jobs which are linked with one another. A manager will have occasions to deal with some network or other. Certain problems pertaining one works are taken up for consideration in this unit.

A network is a series of related activities and events which result in an end product or service. The activities shall follow a prescribed sequence. For example, while constructing a house, laying the foundation should take place before the construction of walls. Fitting water tapes will be done towards the completion of the construction. Such a sequence cannot be altered.

A project network consists of a finite number of events and activities, by adhering to a certain specified sequence. There shall be a **start event** and an **end event (or stop event)**. All the other events shall be between the start and the end events. The activities shall be marked by directed arrows. An activity takes the project from one event to another event.

An event takes place at a point of time where as an activity takes place from one point of time to another point of time.

9.3. Solved Problem

Problem1:

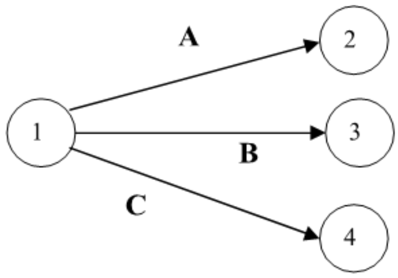
Construct the network diagram for a project with the following activities**:**

Activity	Name of	Immediate
Event	Activity	Predecessor Activity
$1 \rightarrow 2$	A	-
$1 \rightarrow 3$	B	-
$1 \rightarrow 4$	C	-
$2 \rightarrow 5$	D	A
$3 \rightarrow 6$	E	B
$4 \rightarrow 6$	F	C
$5 \rightarrow 6$	G	D

Solution:

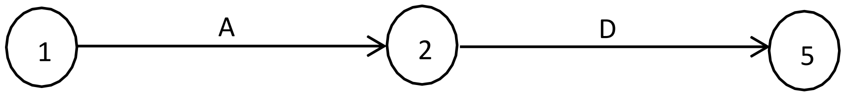
The start event is node1.

The activities A,B,C start from node1and none of them has a predecessor activity. A joins nodes1and2; B joins nodes1and3;C joins nodes1and 4.So we get the following:

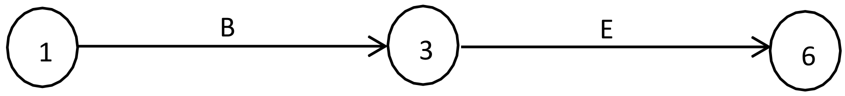


This is a part of the network diagram that is being constructed.

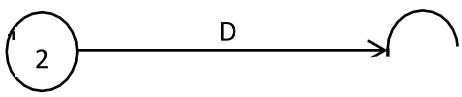
Next, activity D has A as the predecessor activity. D joins nodes 2 and 5.So we get
Next, activity E has B as the predecessor activity. E joins nodes 3 and 6.



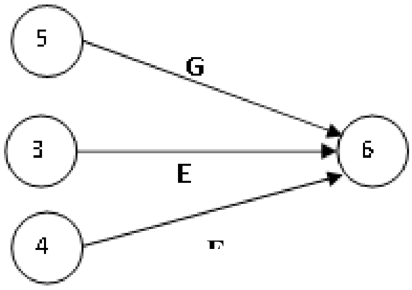
So we get



Next, activity G has D as the predecessor activity. G joins nodes 5 and 6. Thus we obtain

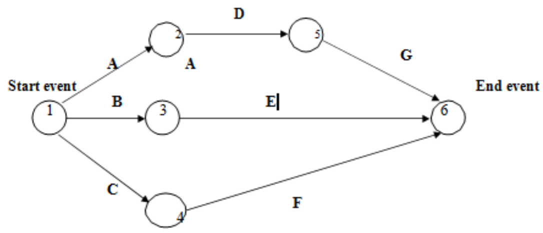


Since activities E,F,G terminate in node 6,weget



6 is the end event.

Combining all the pieces together, the following network diagram is obtained for the given project



We validate the diagram by checking with the given data.

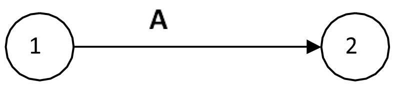
Problem2:

Develop a network diagram for the project specified below:

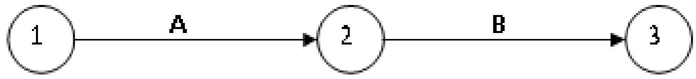
Activity	Immediate Predecessor Activity
A	-
B	A
C,D	B
E	C
F	D
G	E,F

Solution:

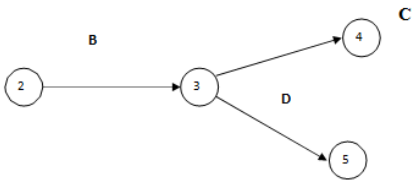
Activity A has no predecessor activity. i.e., It is the first activity. Let us suppose that activity A takes the project from event 1 to event 2. Then we have the following representation for A:



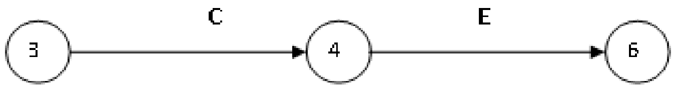
For activity B, the predecessor activity is A. Let us suppose that B joins nodes 2 and 3. Thus we get



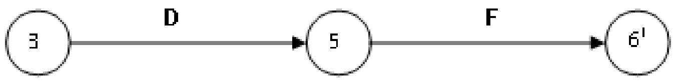
Activities C and D have B as the predecessor activity. Therefore, we obtain the following:



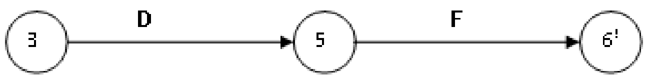
Activity E has D as the predecessor activity. So we get



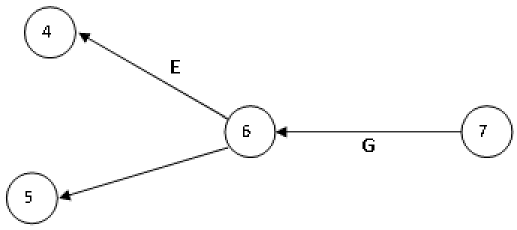
Activity F has D as the predecessor activity. So we get



Activity G has E and F as predecessor activities. This is possible only if nodes 6 and 6' are one and the same. So, rename node 6' as node 6.Then we get

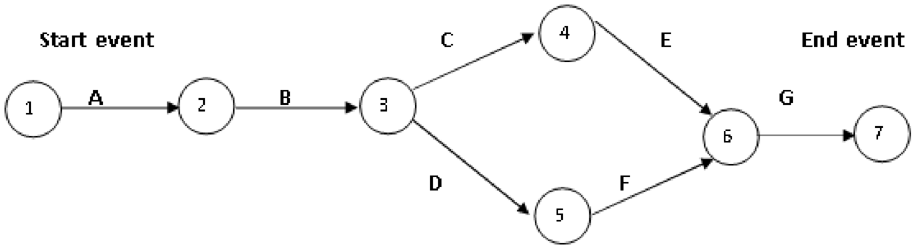


and



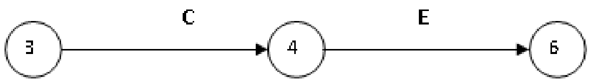
G is the last activity.

Putting all the pieces together, we obtain the following diagram the project network:

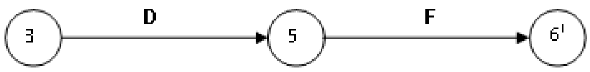


The diagram is validated by referring to the given data.

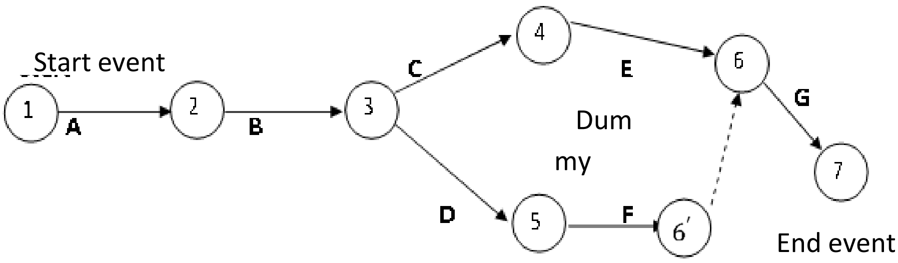
Note: An important point may be observed for the above diagram. Consider the following parts in the diagram



and



We took nodes 6 and 6' as one and the same. Instead, we can retain them as different nodes. Then, in order to provide connectivity to the network, we join nodes 6' and 6 by a dummy activity. Then we arrive at the following diagram for the project network:



Let us Sum Up

In this unit, the students have learned to understand basic terminologies and Project Network of Network models.

Check your Progress

1. In program evaluation review technique network each activity time assume a beta distribution because …………….
   a. it is a unimodal distribution that provides information regarding the uncertainty of time estimates of activities
   b. it has got finite non-negative error
   c. it need not be symmetrical about model value
   d. the project is progressing well
2. If there is no non-negative replacement ratio in solving a linear programming problem then the solution is …………..
   a. feasible
   b. bounded
   c. unbounded
   d. infinite
3. The objective of network analysis is to …………….
   a. minimize total project duration
   b. minimize total project cost
   c. minimize production delays, interruption and conflicts
   d. maximize total project duration

Glossary

Predecessor Event: The event just before another event is called the predecessor event.

Successor Event: The event just following another event is called the successor event.

Project network: A finite number of events and activities, by adhering to a certain specified sequence.

Answers to check your progress

1. a

2. c

3. a

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008
3. Gupta. P. K, Man Mohan, Kanti Swarup: “Operations Research”, Sultan Chand, 2008.

Unit-10

Network Models

STRUCTURE

Overview

Networking is an important concept where by the shortest path is identified and the optimal solution is obtained. The methodology is explained below.

Objectives

After studying this unit, the students will be able to understand

- Network Models and its techniques.

10.1. Introduction

Network models are possibly still the most important of the special structures in linear programming. The characteristics of network models were examined and formulated some examples of these models, and give one approach to their solution. The approach presented here is simply derived from specializing the rules of the simplex method to take advantage of the structure of network models. The resulting algorithms are extremely efficient and permit the solution of network models so large that they would be impossible to solve by ordinary linear-programming procedures. Their efficiency stems from the fact that a pivot operation for the simplex method can be carried out by simple addition and subtraction without the need for maintaining and updating the usual tableau at each iteration. Further, an added benefit of these algorithms is that the optimal solutions generated turn out to be integer if the relevant constraint data are integer.

10.2. Network Models

A Network Model is one which can be represented by a set of nodes, a set of arcs, and functions (e.g. costs, supplies, demands, etc.) associated with the arcs and/or nodes. The network model is graphical in that it is presented as a collection of the nodes and arcs drawn in the figure. The nodes represent the cities of this problem, and we name them with the shortened names of the supply and demand cities. Arcs are the directed line segments of the figure.

Terminology

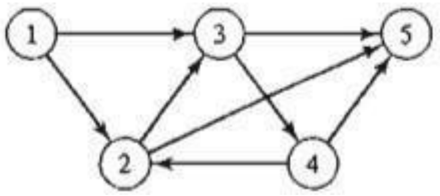
- Node (vertex)
- Arc (link, edge, or branch): Weight (Wij)
- Graph – Network
- Directed network (all arcs are directed) vs. undirected network (all arcs are undirected).
- Directed path from node *i*to node *j* (flow from node *i*to node *j* along this path is feasible) vs. undirected path (flow can violate the direction of arcs)
- Cycle: A path that begins and ends at the same node.
- Tree is a connected network (for some subset of nodes) that contains no undirected cycles.
- Spanning tree—is a connected network for all *n* nodes that contains no undirected cycles.

Network Definitions. A network consists of a set of nodes linked by arcs (or branches). The notation for describing a network is (N, A), where N is the set of nodes and A is the set of arcs. As an illustration, the network in Figure 6.1 is described as

N = {1,2,3,4,5}

A = {(1,2), (1,3), (2,3), (2,5), (3,4), (3,5), (4,2), (4,5)}

Associated with each network is a flow (e.g., oil products flow in a pipeline and automobile traffic flows in highways). In general, the flow in a network is limited by the capacity of its arcs, which may be finite or infinite.



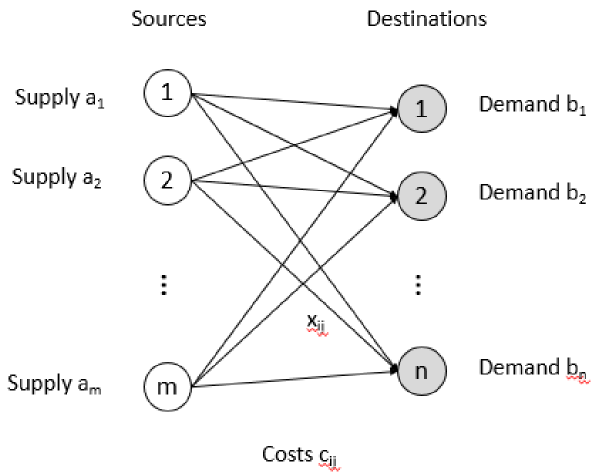
10.3. Techniques of Network Models

Techniques of Network Models or Algorithms

- Transportation
- Assignment
- Minimal Spanning Tree
- Shortest Path Algorithms
- Maximal Flow Problems or Algorithms
- Minimum Cost Flow
- Critical Path Method (CPM)

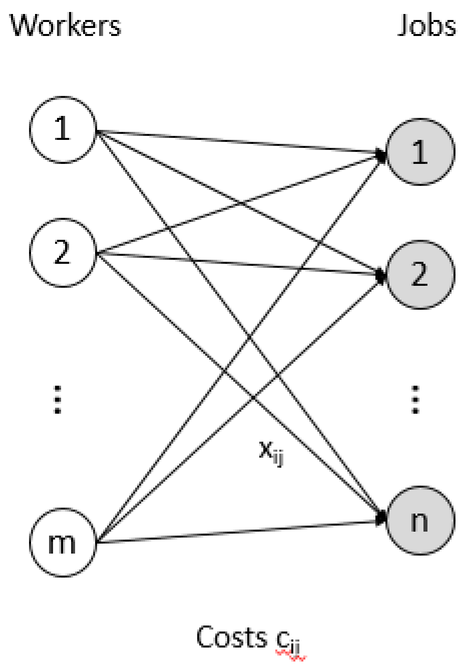
Transportation problem

•Transportation problem is to minimize the total shipping costs of transporting goods from *m* origins or sources (each with a supply *ai*) to *n* destinations (each with a demand *bj*), when the unit shipping cost from source, *i*, to a destination, *j*, is *cij*



Assignment Problem

- An Assignment Problem is to minimize the total cost assignment of *m* workers to*n* jobs, given that the cost of worker *I* performing job *j* is *cij.*



Minimal Spanning Tree

- Problem Description: In a connected, undirected graph, a spanning tree, is a subgraph that connects all nodes on the network.
- Definition: A network model is designed by inserting enough arcs to satisfy the requirement that there be a path between every pair of nodes is called Minimal Spanning Tree
  - Weights on the arcs: Costs
  - The graph: undirected
  - Network Model
- Objective Function: Minimize cost for the total length of the arcs from the origin to the destination
- Constraint: Path through a network
- Decision Variable: Path through a network
- Algorithm: Kruskal's algorithm, Prim's Algorithm, Greedy methods.
- Application: How to install phone lines under the roads to establish telephone communication among all the stations in the smallest cost

Shortest Path Algorithms

- Problem Description: Given a network with distance (or travel time, or cost, etc.) associated with each arc, so find a path is important through the network
- Definition: The problem of finding the shortest path from the origin to the destination in the network is called a shortest path problem.
- Objective: To find the shortest path from the origin to the destination
- Weights on the arcs: Distance or Costs or Time
- The graph: directed
- Network Model
- Objective Function: Minimize cost or time for the flow from the origin to the destination
- Constraint: Flow through a network
- Decision Variable: Flow through a network
- Algorithm: Dijkstra's algorithm, Floyd's Algorithm
- Application: Determine which route from the park entrance (O) to station T has the smallest total distance

Maximal Flow Problems or Algorithms

- Problem Description: A network model has a capacity that limits the quantity of a product that may be transported through the arc. So the flow of material is important.
- Definition: The problem of transporting the maximum amount of flow from the origin to the destination in the network is called a maximum flow problems.
- Weights on the arcs: Capacities (Material, Product)
- The graph: directed
- Network Model
- Objective Function: Maximize the amount of material sent from the origin to the destination
- Constraint: Flow of material
- Decision Variable: The amount of material
- Algorithm: Max Flow – Min Cut, Augmenting Path Algorithm

- Application: How to route various trips to maximize the number of trips that can be made per day without violating the limits

Let us Sum Up

In this unit, the students have learned to understand Network Models and its techniques.

Check your Progress

1. Please state which statement is true.

(i) All linear programming problems may not have unique solutions

(ii) The artificial variable technique is not a device that does not get the starting basic feasible solution.

a) Both (i) and(ii)

b) (ii) only

c) (i) only

d) Both are incorrect

2. Please state which statement is incorrect.

(i) Linear programming was first formulated by an English economist L.V. Kantorovich

(ii) LP is generally used in solving maximization or minimization problems subject to certain assumptions.

a) (ii) only

b) (i) only

c) Both (i) and (ii)

d) Both are correct

3. ------------ which is a subclass of a linear programming problem (LPP)

a) Programming problem b) Transportation problem c) Computer problem d) Both are incorrect

Glossary

Network Model: One which can be represented by a set of nodes, a set of arcs, and functions (e.g. costs, supplies, demands, etc.) associated with the arcs and/or nodes.

Answers to check your progress

1.C

2.B

3.C

Suggested readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008
3. Gupta. P. K, Man Mohan, Kanti Swarup: “Operations Research”, Sultan Chand, 2008.

Unit-11

Critical Path method (CPM)

STRUCTURE

Overview

In this unit we are going to deal with the critical path method to get the optimal cost of a problem.

Objectives

After studying this unit, the students will be able to solve Critical Path Method (CPM)

11.1. Critical Path Method (CPM)

The Critical Path Method (CPM) aims at the determination of the time to complete a project and the important activities on which a manager shall focus attention.

Project Completion Time

From the start event to the end event, the time required to complete all the activities of the project in the specified sequence is known as the project completion time.

Path in a Project

A continuous sequence, consisting of nodes and activities alternatively, beginning with the start event and stopping at the end event of a network is called a path in the network.

Critical Path and Critical Activities

The path with the largest time is called the **critical path** and the activities along this path are called the **critical activities** or **bottle neck activities**.

Solved Problems Problem1:

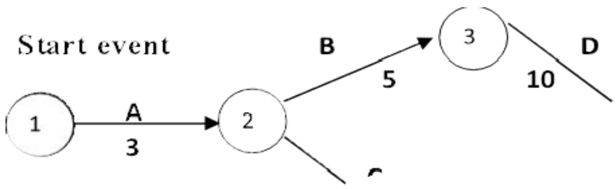
The following details are available regarding a project:

Activity	Predecessor Activity	Duration (Weeks)
A	-	3
B	A	5
C	A	7
D	B	10
E	C	5
F	D,E	4

Determine the critical path, the critical activities and the project completion time.

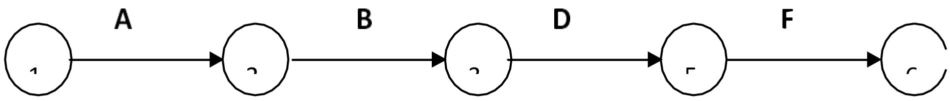
Solution:

First let us construct the network diagram for the given project. We mark the time estimates along the arrows representing the activities.



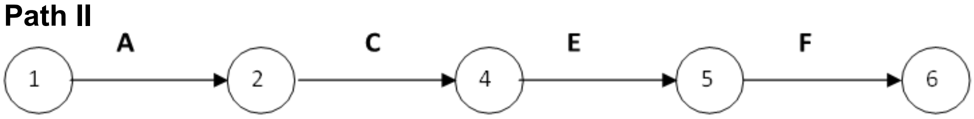
Consider the paths, beginning with the start node and stopping with the end node. There are two such paths for the given project. They are as follows:

Path I



With a time of 3+5+10+4=22weeks.

Path II



With a time of3+7+5+4=19weeks.

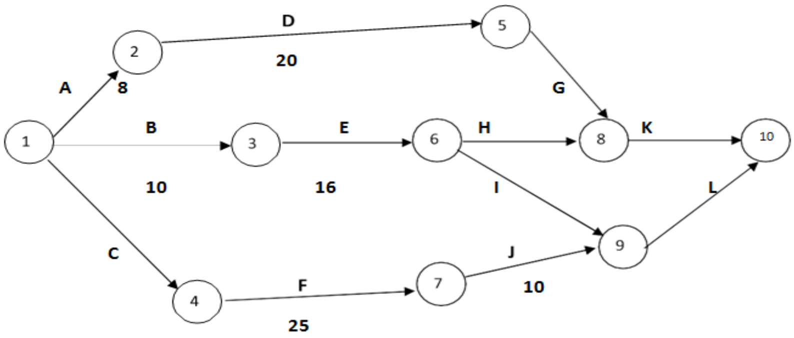
Compare the times for the two paths. Maximum of {22,19} = 22. We see that path I has the maximum time of 22 weeks. Therefore, path I is the critical path. The critical activities are A,B,D and F. The project completion timeis22weeks.

WenoticethatCandEarenon-criticalactivities.TimeforpathI-Timeforpath II= 22-19=3weeks.

Therefore, together the non- critical activities can be delayed up to a maximum of 3weeks, without delaying the completion of the whole project.

Problem2:

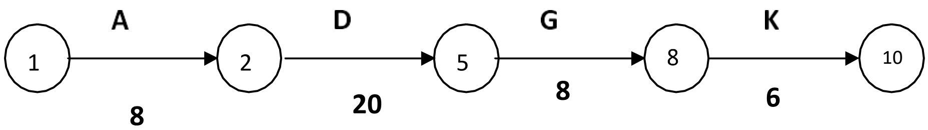
Find out the completion time and the critical activities for the following project:



Solution:

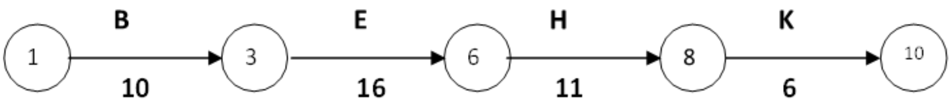
In all, we identify 4 paths, beginning with the start node of 1 and terminating at the end node of 10. They are as follows:

Path I



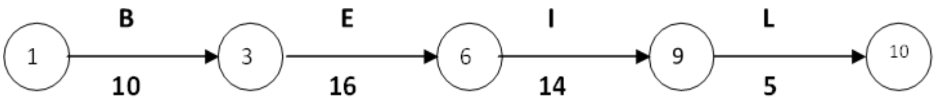
Time for the path **=** 8+20+8+6 = 42 units of time.

Path II



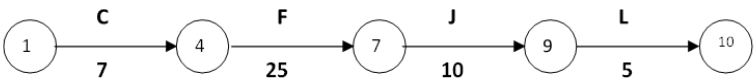
Time for the path **=** 10+16+11+6 = 43units of time.

Path III



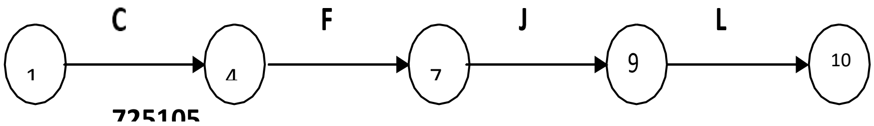
Time for the path**=**10+16+14+5=45 units of time.

Path IV



Time for the path **=** 7+25+ 10+ 5 = 47 units of time.

Compare the times for the four paths. Maximum of {42,43,45,47} = 47.



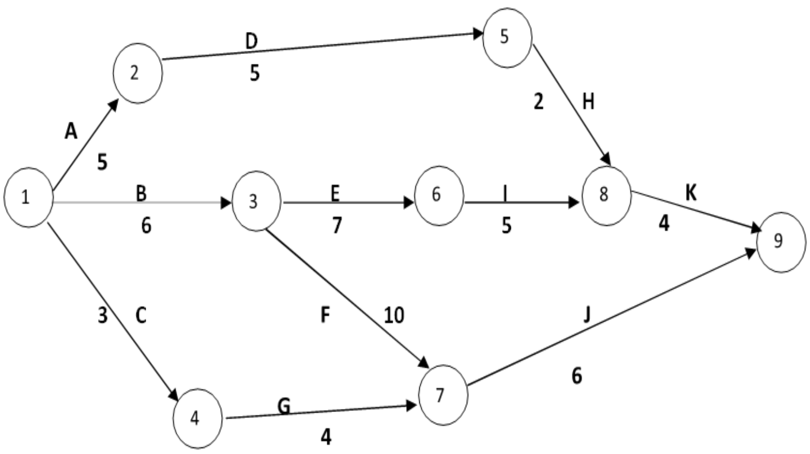
The critical activities are C,F,J and L. The non-critical activities are A,B,D,E,G,H,I and K. The project completion time is 47 units of time.

Problem3:

Draw the network diagram and determine the critical path for the following project:

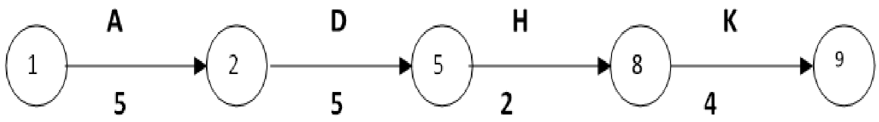
Activity	Time estimate (Weeks)
1-2	5
1-3	6
1- 4	3
2 -5	5
3 -6	7
3 -7	10
4 -7	4
5 -8	2
6 -8	5
7 -9	6
8 -9	4

Solution: We have the following network diagram for the project:



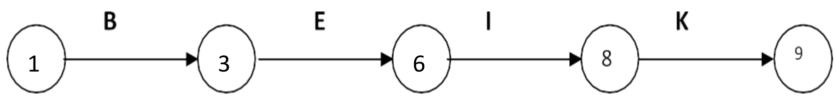
Solution:

Path I



Time for the path **=** 5+5+2+4 = 16 weeks.

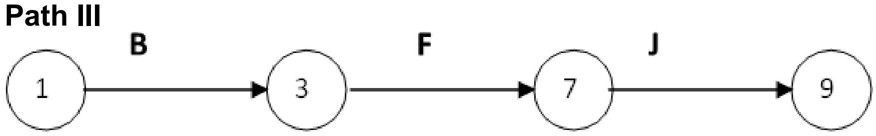
Path II



6 7 5 4

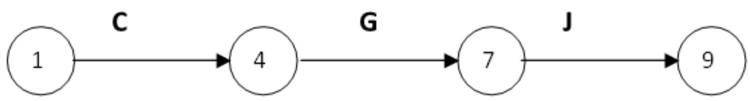
Time for the path **=** 6+7+5+4 = 22weeks.

Path III



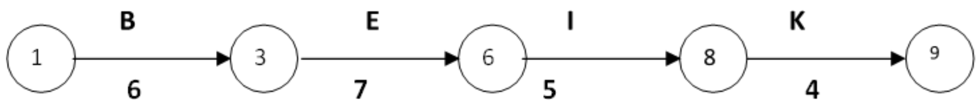
Time for the path **=** 6+10+6 =16 weeks.

Path IV



Time for the path **=** 3+4+6 = 13 weeks.

Compare the times for the four paths. Maximum of {16,22,16,13} = 22. We see that the following path has the maximum time and so it is the critical path:



The critical activities are B,E,I and K. The non-critical activities are A,C,D,F,G,H and J. The project completion time is 22 weeks.

Let us Sum Up

In this unit, the students have learned to solve Critical Path Method (CPM).

Check your progress

1. In PERT chart, the activity time distribution is
   a. Normal
   b. Binomial
   c. Poisson
   d. Beta
2. Critical path method is good for
   a. Small project only
   b. Large project only
   c. Both small and large project
   d. None of the above
3. In construction project planning, free float can affect which of the following?
   a. Only that particular activity
   b. Succeeding activity
   c. Overall completion
   d. Preceding activity

Glossary

Path: A continuous sequence, consisting of nodes and activities alternatively, beginning with the start event and stopping at the end event of a network is called a path in the network.

Critical Path Method (CPM): The method for determination of the time to complete a project and the important activities on which a manager shall focus attention.

Answers to check your progress

1.d

2.b

3.d

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.
2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008
3. Gupta. P. K, Man Mohan, Kanti Swarup: “Operations Research”, Sultan Chand, 2008.

Unit-12

Program Evaluation Review Technique (PERT)

STRUCTURE

Overview

It is used to help in project planning and control. The project manager can monitor the project continuously and take necessary steps to get the optimal cost.

Objectives

After studying this unit, the students will be able:

- to solve Programme Evaluation and Review Technique (PERT)

12.1. Program Evaluation Review Technique (PERT)

Programme Evaluation and Review Technique (**PERT**) is a tool that would help a project manager in project planning and control. It would enable him in continuously monitoring a project and taking corrective measures wherever necessary. This technique involves statistical methods.

Note that in CPM, the assumption is that precise time estimate is available for each activity in a project. However, one finds most of the times that this is not practically possible.

In PERT, we assume that it is not possible to have precise time estimate for each activity and instead, probabilistic estimates of time alone responsible. A multiple time estimate approach is followed here. In probabilistic time estimate, the following 3 types of estimate are possible:

1. Pessimistic time estimate (*tp*)
2. Optimistic time estimate (*to*)
3. Most likely time estimate (*tm*)

The optimistic estimate of time is based on the assumption that an activity will not involve any difficulty during execution and it can be completed within a short period. On the other hand, a pessimistic estimate is made on the assumption that there would be unexpected problems during the execution of an activity and hence it would consume more time. The most likely time estimate is made in between the optimistic and the pessimistic estimates of time.

Thus the three estimates of time have the relationship

to≤tm≤tp.

Practically speaking, neither the pessimistic nor the optimistic estimate may hold in reality and it is the most likely time estimate that is expected to prevail in almost all cases. Therefore, it is preferable to give more weight to the most likely time estimate.

We give a weight of 4 to most lik~~ely time est~~imate and a weight of 1 each to the pessimistic and optimistic time estimates. We arrive at a time estimate (*te*) as the weighted average of these estimates as follows:

t_e =t_o + 4 + t_p 6

Since we have taken 6units (1for *tp*,4 for *tm* and 1for *to*), we divide the sum by 6. With this time estimate, we can determine the project completion time as applicable for CPM.

Since PERT involves the average of three estimates of time for each activity, this method is very practical and the results from PERT will behave a reasonable amount of reliability.

Measure of Certainty

The 3 estimates of time are such that

to≤tm≤tp. Therefore the range for the time estimate is $t_p - t_o$.

The time taken by an activity in a project network follows a distribution with a standard deviation of one sixth of the range, approximately.
i.e., The standard deviation = σ =$\frac{tp-to}{}$and the 6
variance= $\sigma^2 \underline{tp-to}^2$= ()6

The certainty of the time estimate of an activity can be analysed with the help of the variance. The greater the variance, the more uncertainty in the time estimate of an activity.

12.2. Solved Problems

Problem1:

Two experts A and B examined an activity and arrived at the following time estimates.

	to	*Tm*	*tp*
A	4	6	8
B	4	7	10

Determine which expert is more certain a both is estimates of time:

Solution:

Variance(σ^2)in time estimates$\underline{tp-}$= ()6

In the case of expert A, the variance8−4 ^{2}4= () =66

As regards expert B, the variance

10−4 ^{2}= () = 16

So, the variance is less in the case of A. Hence, it is concluded that the expert A is more certain a both is estimates of time.

Problem2:

Find out the time required to complete the following project and the critical activities:

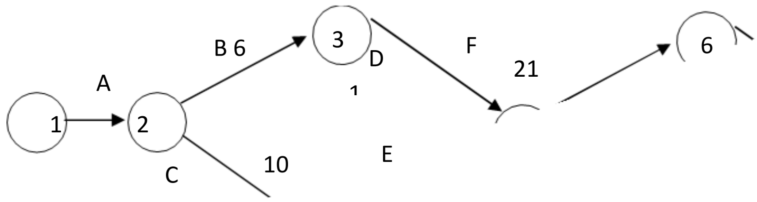
Activity	Predecessor Activity	Optimistic time Estimate (to days)	Most likely time estimate (tm days)	Pessimistic Time estimate (tpdays)
A	-	2	4	6
B	A	3	6	9
C	A	8	10	12
D	B	9	12	15
E	C	8	9	10
F	D,E	16	21	26
G	D,E	19	22	25
H	F	2	5	8
I	G	1	3	5

Solution:

From the three time estimates *tp*, *tm* and *to*, calculate *te* for each activity. We obtain the following table:

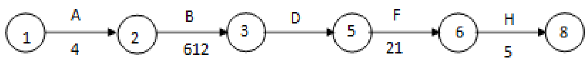
Activity	Optimistic time estimate (to)	4 x Most likely time estimate	Pessimistic time estimate(tp)	to+4tm +tp	Time estimate $t_e t_o + 4t_m + t_p$=6
A	2	16	6	24	4
B	3	24	9	36	6
C	8	40	12	60	10
D	9	48	15	72	12
E	8	36	10	54	9
F	16	84	26	126	21
G	19	88	25	132	22
H	2	20	8	30	5
I	1	12	5	18	3

Using the single time estimates of the activities, we get the following network diagram for the project.

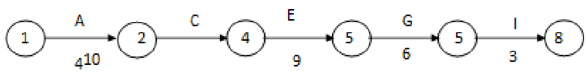


Consider the paths, beginning with the start node and stopping with the end node. There are four such paths for the given project. They are as follows:

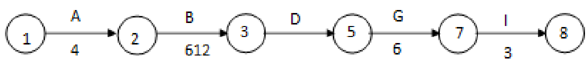
Path I



Time for the path: 4+6+12+21+5 = 48days.

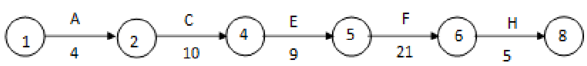


Path II



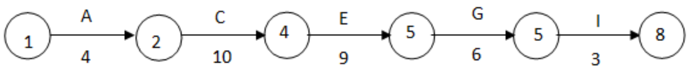
Time for the path: 4+6+12+6+3 = 31days.

Path III



Time for the path: 4+10+9+21+5 = 49days.

Path IV



Time for the path: 4+10+9+6+3 = 32days. Compare the times for the four paths.

Maximum of {48,31,49,32} = 49.

We see that Path III has the maximum time.

Therefore the critical path is Path III. i.e.,1→2→4→5→6→8. The critical activities are A,C, E, F and H.

The non-critical activities are B, D, G and I. Project time (Also called project length) = 49 days.

Problem 3:

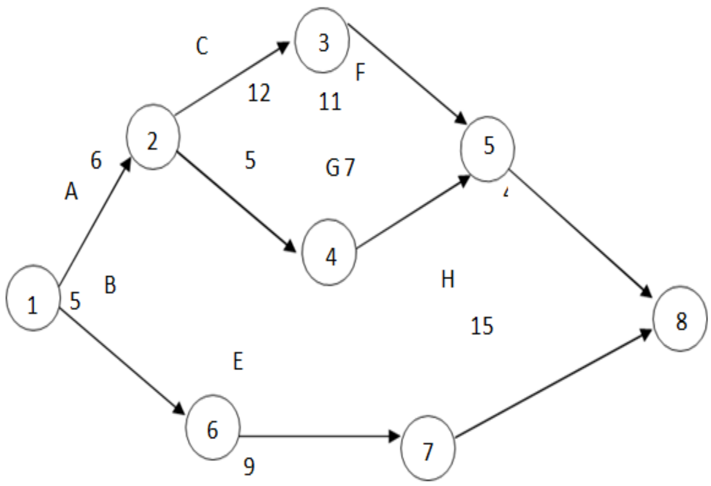
Find out the time, variance and standard deviation of the project with the following time estimates in weeks:

Activity	Optimistic time estimate(to)	Most likely time estimate(tm)	Pessimistic time estimate(tp)
1-2	3	6	9
1-6	2	5	8
2-3	6	12	18
2-4	4	5	6
3-5	8	11	14
4-5	3	7	11
6-7	3	9	15
5-8	2	4	6
7-8	8	16	18

Solution:

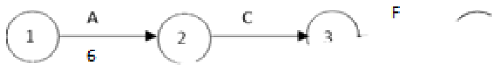
We obtain the following table:

Activity	Optimistic time estimate (to)	4xMost likely time estimate	Pessimistic time estimate(tp)	to+4tm+tp	Time estimate t_e $t_o + 4t_m + t_p$ =6
1-2	3	24	9	36	6
1-6	2	20	8	30	5
2-3	6	48	18	72	12
2-4	4	20	6	30	5
3-5	8	44	14	66	11
4-5	3	28	11	42	7
6-7	3	36	15	54	9
5-8	2	16	6	24	4
7-8	8	64	18	90	15



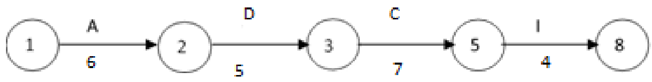
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



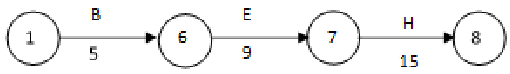
Time for the path : 6+12+11+4 = 33 weeks.

Path II



Time for the path : 6+5+7+4 = 22 weeks.

Path III



Time for the path: 5+9+15 = 29 weeks.

Compare the times for the three paths. Maximum of {33,22,29} = 33.

It is noticed that Path I has the maximum time.

Therefore the critical path is Path I .i.e., 1 → 2 → 3 → 5 → 8 The critical activities are A,C,F and I.

The non-critical activities are B, D, G and H. Project time = 33weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic Time estimate(to)	Most likely Time estimate(tm)	Pessimistic Time estimate(tp)	Range ($t_p - t_o$)	Standard deviation= $\underline{tp-to}$ σ =6	Variance $t_p - t_o{}^2\sigma^2$ = () 6
A:1→2	3	6	9	6	1	1
C:2→3	6	12	18	12	2	4
F:3→5	8	11	14	6	1	1
I:5→8	2	4	6	4	2/3	4/9

Variance of project time

(Also called Variance of project length) = Sum of the variances for the critical activities

=1+4+1+4/9 =58/9 Weeks.

Standard deviation of project time =√Variance

=√58/9

=2.54weeks.

Problem 4

A project consists of seven activities with the following time estimates. Find the probability that the project will be completed in 30weeks or less.

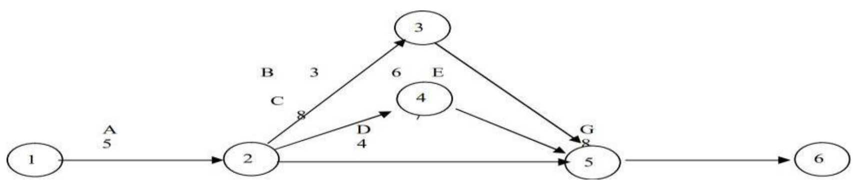
Activity	Predecessor Activity	Optimistic time Estimate (to days)	Most likely time estimate(tm days)	Pessimistic time estimate(tp days)
A	-	2	5	8
B	A	2	3	4
C	A	6	8	10
D	A	2	4	6
E	B	2	6	10
F	C	6	7	8
G	D,E,F	6	8	10

Solution:

From the three time estimates *tp*,*tm* and *to*, calculate *te* for each activity.

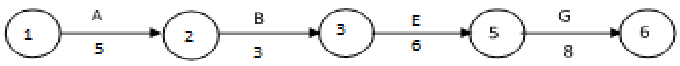
Activity	Optimistic time estimate (to)	4 x Most likely time estimate	Pessimistic time estimate (tp)	$t_o + 4t_m + t_p$	Time estimate $t_e = \frac{t_o + 4t_m + t_p}{6}$
A	2	20	8	30	5
B	2	12	4	18	3
C	6	32	10	48	8
D	2	16	6	24	4
E	2	24	10	36	6
F	6	28	8	42	7
G	6	32	10	48	8

With the single time estimates of the activities, the following network diagram is constructed for the project.



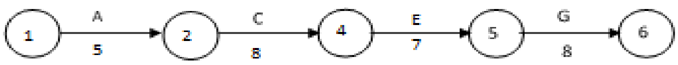
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



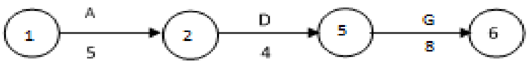
Time for the path: 5+3+6+8 = 22 weeks.

Path II



Time for the path: 5+8+7+8 = 28 weeks.

Part III



Time for the path:5+4+8=17 weeks. Compare the times for the three paths. Maximum of {22,28,17}=28.

It is noticed that Path II has the maximum time. Therefore the critical path is Path II.i.e.,1→2→4→5→6. The critical activities are A,C,F and G.

The non-critical activities are B,D and E. Project time = 28weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (to)	Most likely time estimate (tm)	Pessimistic time estimate (tp)	Range (tp-to)	Standard deviation= $\underline{tp-to}$ σ =6	Variance $t_p - t_o^2$ σ^2 = (
A:1→2	2	5	8	6	1	1
C:2→4	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$
F:4→5	6	7	8	2	$\frac{1}{3}$	$\frac{1}{9}$
G:5→6	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$

Standard deviation of the critical path = √2 = 1.414. The standard normal variate is given by the formula:

$$Z = \ value\ of\ t - Expected\ value\ of\ t \text{ in } the\ critical\ path\ SD\ for\ the\ critical\ path$$

So we get Z = $^{30-28}$ =. 1.414

1.414

We refer to the Normal Probability Distribution Table. Corresponding to Z=1.414, we obtain the value of 0.4207 we get 0.5+0.4207 = 0. 9207

Therefore the required probability is 0.92. i.e., there is 92 % chance that the project will be completed before 30 weeks. In other words, the chance that it will be delayed beyond30 weeks is 8%

Let us Sum Up

In this unit, the students have learned to solve Programme Evaluation and Review Technique (PERT).

Check your progress

1. A feasible solution is called a basic feasible solution if the number of non-negative allocations is equal to ____________

a. M-n+1

b. m-n-1

c. m+n-1

d. None of the above

2. Any feasible solution to a transportation problem containing m origins and n destinations is said to be ___________

   a. Independent

   b. Degenerate

   c. Non-degenerate

   d. Both A and B

3. A path formed by allowing horizontal and vertical lines and the entire corner cells of which are occupied is called a ______________

   a. Occupied path

   b. Open path

   c. Closed path

   d. None of the above

Glossary

Pessimistic time estimate (*tp*): The optimistic estimate of time is based on the assumption that an activity will not involve any difficulty during execution and it can be completed within a short period

Optimistic time estimate (*to*): The pessimistic estimate is made on the assumption that there would be unexpected problems during the execution of an activity and hence it would consume more time.

Most likely time estimate (*tm*): The most likely time estimate is made in between the optimistic and the pessimistic estimates of time. Thus the three estimates of time have the relationship.

Answer to check your progress

1. c

2.c

3.c

Suggested Readings

1. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.

2. Sankar P. Iyer, Operations Research, Tata McGraw-Hill Education, 2008

3. Gupta. P. K, Man Mohan, Kanti Swarup: “Operations Research”, Sultan Chand, 2008.

Block-4: Introduction

Block-4: Game Theory has been divided in to Four Units. **Unit-13: Zero-sum games and Non-zero sum games –** Introduction, Basic Concepts, Example

Unit-14: Pure & Mixed Strategy- Introduction, Pure & Mixed Strategy

Unit-15: Maximin-Minimax Principle - Maximin–Minimax Principle, Solved Problems.

Unit-16: Dominance Property - Dominance Property, Solved Problems

In all the units of Block -4 **Game Theory**, the Check your progress, Glossary, Answers to Check your progress and Suggested Reading has been provided and the Learners are expected to attempt all the Check your progress as part of study.

Unit -13

Zero-sum games and Non-zero sum games

STRUCTURE

Overview

In this unit we are going to focus on the mathematical framework for analyzing the decision making processes and strategies of adversaries i.e. the players. Simple strategy involved are noted down to attain the solution.

Objectives

After studying this unit, the students will be able:

- to understand the Zero-sum games and Non- zero sum games.

13.1. Introduction

Game theory provides a mathematical framework for analysing the decision-making processes and strategies of adversaries (or *players*) in different types of competitive situations. The simplest type of competitive situations is *two-person, zero-sum games*. These games involve only two players; they are called *zero-sum* games because one player wins whatever the other player loses. As we have seen, the Nash equilibrium and the maximin are two different concepts that reflect different behavioural aspects: the first is an expression of stability, while the second captures the notion of security. Despite the different roots of the two concepts, there are cases in which both lead to the same results. A special case where this occurs is in the class of two- player zero-sum games, which is the subject of this section. A two-player game is called a *zero- sum* game if the sum of the payoffs to each player is constant for all possible outcomes of the game. More specifically, the terms (or coordinates) in each payoff vector must add up to the same value for each payoff vector. Such games are sometimes called *constant-sum* games instead.

A two-player game is a zero-sum game if for each pair of strategies (s_I, s_{II}) one has

$$(s_I,\ s_{II}) + (s_{I},) = 0.$$

In other words, a two-player game is a zero-sum game if it is a closed system from the perspective of the payoffs: each player gains what the other player loses. It is clear that in such a game the two players have diametrically opposed interests.

13.2. Basic Concepts

This game of odds and evens illustrates important concepts of simple games.

- A two-person game is characterized by the strategies of each player and the payoff matrix.
- The payoff matrix shows the gain (positive or negative) for player 1 that would result from each combination of strategies for the two players. Note that the matrix for player 2 is the negative of the matrix for player 1 in a zero-sum game.
- The entries in the payoff matrix can be in any units as long as they represent the utility (or value) to the player.
- There are two key assumptions about the behavior of the players. The first is that both players are rational. The second is that both players are greedy meaning that they choose their strategies in their own interest (to promote their own wealth).

13.3. Example

		Player II			
		L	*C*	*R*	$\min_{s_{II} \in S_{II}} u_I(s_I, s_{II})$
	T	3, −3	−5, 5	−2, 2	−5
Player I	*M*	1, −1	4, −4	1, −1	1
	B	6, −6	−3, 3	−5, 5	−5
$\min_{s_I \in S_I} u_{II}(s_I, s_{II})$		−6	−4	−1	1, −1

Example 13.1 Consider the two-player game appearing in Figure 13.1.

Figure 13.1 A two-player zero-sum game

In this example, $v_I = 1$ and $v_{II} = -1$. The maxmin strategy of Player I is M and that of Player II is R. The strategy pair (M, R) is also the equilibrium of this game (check!). In other words, here we have a case where the vector of maxmin strategies is also an equilibrium point: the two concepts lead to the same result.

In the game in Example 13.1, for each pair of strategies the sum of the payoffs that the two players receive is zero. In other words, in any possible outcome of the game the payoff one player receives is exactly equal to the payoff the other player has to pay.

		Player II		
		L	*C*	*R*
	T	3	−5	−2
Player I	*M*	1	4	1
	B	6	−3	−5

Figure 13.2 The payoff function u of the zero-sum game in Example 13.1

The study of two-player zero-sum games. Since the payoffs $uI$and uIIsatisfy

$u_I + u_{II} = 0$, (13.20)

we can confine our attention to one function, $u_I = u$, with $u_{II} = -u$. The function u will be termed the payoff function of the game, and it represents the payment that Player II makes to Player I. Note that this creates an artificial asymmetry (albeit only with respect to the symbols being used) between the two players: Player I, who is usually the row player, seeks to maximize u(s) (his payoff) and Player II, who is usually the column player, is trying to minimize u(s), which is what he is paying (since his payoff is −u(s)).

The game in Example 13.1 can therefore be represented as shown in Figure 13.2.

The game of Matching Pennies can also be represented as a zero-sum game (see Figure 13.3).

		Player II	
		H	T
Player I	H	1	−1
	T	−1	1

Figure 13.3 The payoff function u of the game Matching Pennies

Consider now the maxmin values of the players in a two-player zero-sum game. Player I's maxmin value is given by,

v_I=max min u(s_I , s_{II}),(13.21)

$sI \in SIsII \in SII$

and Player II's maxmin value is

v_I= max min (− u(s_I , s_{II})) = - min max u(s_I , s_{II}) 13.22)

$sII \in SIIsI \in SIsII \in SIIsI \in SI$

		Player II		
		L	R	$\min_{s_{II} \in S_{II}} u_I(s_I, s_{II})$
Player I	T	−2	5	−2
	B	3	0	0
$\max_{s_I \in S_I} u_{II}(s_I, s_{II})$		3	5	0, 3

Figure 13.4 A game in strategic form with the maxmin and minmax values

Denote

$\underline{v}$: =max min u(s_I , s_{II}) (13.23)

$sI \in SIsII \in SII$

: = min max u(s_I , s_{II}) (13.24)

$sII \in SIIsI \in SI$

The value $\underline{v}$is called the maxmin value of the game, and $v$is called the minmax value. Player I can guarantee that he will get at least $\underline{v}$, and Player II can guarantee that he will pay no more than v. A strategy of Player I that guarantee$\underline{v}$is termed a maxmin strategy. A strategy of Player II that guarantees $v$is called a minmax strategy.

We next calculate the maxmin value and minmax value in various examples of games. In Example. 13.1, $\underline{v}$= 1 and v= 1. In other words, Player I can guarantee that he will get a payoff of at least 1 (using the maxmin strategy M), while Player II can guarantee that he will pay at most 1 (by way of the minmax strategy R).

Consider the game shown in Figure 4.35. In this figure we have indicated on the right of each row the minimal payoff that the corresponding strategy of Player I guarantees him. Beneath each column we have indicated the maximal amount that Player II will pay if he implements the corresponding strategy.

In this game $\underline{v}$= 0 but v= 3. Player I cannot guarantee that he will get a payoff higher than 0 (which he can guarantee using his maxmin strategy B) and Player II cannot guarantee that he will pay less than 3 (which he can guarantee using his minmax strategy L).

Finally, look again at the game of Matching Pennies (Figure 13.5)

		Player II		
		H	T	$\min_{s_{II} \in S_{II}} u_I(s_I, s_{II})$
Player I	H	1	−1	−1
	T	−1	1	−1
$\max_{s_I \in S_I} u_{II}(s_I, s_{II})$		1	1	−1, 1

Figure 13.5 Matching Pennies with the maxmin and minmax values

In this game, $\underline{v}$= −1 and v= 1. Neither of the two players can guarantee a result that is better than the loss of one dollar (the strategies H and T of Player I are both maxmin strategies, and the strategies H and T of Player II are both minmax strategies).

As these examples indicate, the maxmin value $\underline{v}$and the minmax value $v$may be unequal, but it is always the case that $\underline{v}\leq v$. The inequality is clear from the definitions of the maxmin and minmax: Player I can guarantee that he will get at least $\underline{v}$, while Player II can guarantee that he will not pay more than v. As the game is a zero-sum game, the inequality $\underline{v}\leq v$must hold.

We wish to stress that these theorems show that in two-player zero-sum games the concept of equilibrium, which is based on stability, and the concept of minmax, which is based on security levels, coincide. If security level considerations are important factors in determining players' behavior, one may expect that the concept of equilibrium will have greater predictive power in two-player zero-sum 116 Strategic-form games games (where equilibrium strategies are also minmax strategies) than in more general games in which the two concepts lead to different predictions regarding players' behavior.

Note that despite the fact that the strategic form of the game is implicitly a simultaneously played game in which each player, in selecting his strategy, does not know the strategy selected by the other player, if the game has a value, then each player can reveal the optimal strategy that he intends to play to the other player and still guarantees his maxmin value. Suppose that s^*_I is an optimal strategy for Player I in a game with value v. Then

$$\min_{s_{II}\in S_{II}} u(s^*_I, s_{II}) = v, \qquad (13.25)$$

and therefore, for each $s_{II} \in s_{II}$ the following inequality is satisfied:

$$u(s^*_I, s_{II}) \geq v. \qquad (13.26)$$

Theorem 13.1.1

If a two-player zero-sum game has a value v, and if s^*_I and s^*_{II} are optimal strategies of the two players, then $s^* = (s^*_I, s^*_{II})$ is an equilibrium with payoff (v, −v).

Proof:

From the fact that both s^*_I and s^*_{II} are optimal strategies, we deduce that

$$u(s^*_I, s_{II}) \geq v, \ \forall s_{II} \in s_{II}, \qquad (13.27)$$

$$u(s_I, s^*_{II}) \leq v, \ \forall s_I \in s_I. \qquad (13.28)$$

Inserting $s_{II} = s^*_{II}$ into Equation (13.27) we deduce $u(s^*_I, s^*_{II}) \geq v$, and inserting $s_I = s^*_I$ into

Equation (13.28) we get $u(s^*_I, s^*_{II}) \leq v$. The equation $v = u(s^*_I, s^*_{II})$ follows. Equations (13.27) and (13.28) can now be written as

$$u(s^*_I, s_{II}) \geq u(s^*_I, s^*_{II}), \ \forall s_{II} \in S_{II}, \qquad (13.29)$$

$$u(s_I, s^*_{II}) \leq u(s^*_I, s^*_{II}), \ \forall s_I \in S_I, \qquad (13.30)$$

and therefore (s^*_I, s^*_{II}) is an equilibrium with payoff (v, −v).

Theorem 13.1.2

If $s^* = (s^*_I, s^*_{II})$ is an equilibrium of a two-player zero-sum game, then the game has a value, v = $u(s^*_I, s^*_{II})$, and the strategies s^*_I and s^*_{II} are optimal strategies.

Proof :

Since (s^*_I, s^*_{II}) is an equilibrium, no player can benefit by a unilateral deviation:

$$u(s_I, s^*_{II}) \leq u(s^*_I, s^*_{II}), \ \forall s_I \in S_I \qquad (13.31)$$

$$u(s^*_I, s_{II}) \geq u(s^*_I, s^*_{II}), \ \forall s_{II} \in S_{II}. \qquad (13.32)$$

Let v = $u(s^*_I, s^*_{II})$. We will prove that v is indeed the value of the game. From Equation (13.31)

we get III

$$u(s^*_I, s_{II}) \geq v, \ \forall s_{II} \in S_{II},$$

(13.33) and therefore v ≥ v. From Equation (4.60) we deduce that

$$u(s_I, s^*_{II}) \leq v, \ \forall s_I \in S_I, \qquad (13.34)$$

		Player II	
		a	*b*
Player I	*A*	1, 1	0, 0
	B	0, 0	3, 3

Figure 13.6 Coordination game

and therefore $v\leq$ v. Because it is always the case that $\underline{v}\leq v$ we get

$$v \leq \underline{v} \leq v \leq v, \quad (13.35)$$

which implies that the value exists and is equal to v. Furthermore, from Equation (13.33) we deduce that s^* is an optimal strategy for Player I, and from Equation (13.34) we deduce that,

s^* is an optimal strategy for Player II.

Let us Sum Up

In this unit, the students have learned to understand the Zero-sum games and Non-zero sum games.

Check your progress

1. General games involve _____________

   a. Single-agent

   b. Multi-agent

   c. Neither Single-agent nor multi-agent

   d. Only Single-agent and multi-agent

2. Adversarial search problems use _____________

   a. Competitive Environment

   b. Cooperative Environment

   c. Neither Competitive nor Cooperative Environment

   d. Only Competitive and Cooperative Environment

3. Mathematical game theory, a branch of economics, views any multi-agent environment as a game provided that the impact of each agent on the others is “significant,” regardless of whether the agents are cooperative or competitive.

   a. True

   b. False

Glossary

Game theory: A mathematical framework for analysing the decision-making processes and strategies of adversaries (or players) in different types of competitive situations.

Zero-sum game: A game involves only two players; they are called zero-sum games because one player wins whatever the other player loses.

Answers to check your progress

1. d

2. a

3. a

Suggested Readings

1. Hamdy A. Taha, Operations Research-An introduction, Pearson Education, 8th Edition / Prentice Hall of India, 2007.

2. Tulsian, P. C., Vishal Pandey, Quantitative Techniques – Theory and Problems, Pearson Publications,2006.

3. A. Ravindren, Don T. Phillips and James J. Solberg, Operations Research Principles and Practice, John Wiley and Sons, 2nd edition, 2000.

Unit -14

Pure and Mixed Strategy

STRUCTURE

Overview

Proper strategy is chosen in a game. A new concept saddle point is introduced in this chapter where maximin = minimax.

Objectives

After studying this unit, the students will be able to understand the Pure & Mixed Strategy

14.1. Introduction

A "saddle point" in a two-person constant-sum game is the outcome that rational players would choose. A saddle point always exists in games of perfect information but may or may not exist in games of imperfect information. By choosing a strategy associated with this outcome, each player obtains an amount at least equal to his payoff at that outcome, no matter what the other player does. This payoff is called the value of the game; as in perfect-information games, it is preordained by the players' choices of strategies associated with the saddle point, making such games strictly determined. A rectangular game without a saddle point cannot be solved using pure strategies or with minimax and Maximin criterion of optimality. So, in this case the best strategies are the mixed strategies. In this strategy the probability with which each action should be selected is calculated. The following theorem is followed to find the solution of a two person 2X2 pay off matrix without a saddle point.

14.2. Pure & Mixed Strategy

Usually, player in game theory uses two types of strategy namely pure strategy and mixed strategy.

Pure Strategy

Particular course of action that are selected by player is called pure strategy (course of action). i.e. each player knows in advance of all strategies out of which he always selects only one particular strategy regardless of the other players strategy, and objective of the player is to maximize gain or minimize loss.

Pure Strategy is based on the following criteria.

- Maximin = Minimax
- Saddle Point exist
- Value of game exist
- Method: Maximin–Minimax Principle, Dominance Property

Mixed Strategy:

Course of action that are to be selected on a particular occasion with some fixed probabilities are called mixed strategies. i.e there is a probabilistic situation and objective of the players is to maximize expected gain or minimize expected losses by making choice among pure strategy with fixed probabilities.

In mixed strategy, If there are 'n' number of pure strategies of the player, there exist a set S={p1,p2,. pn} where pj is the probability with which the pure strategy, j would be selected and whose sum is unity. i.e p1+p2+.....+pn=1 and pj>= 0 for all j=1,2,. n.

In game theory, Strategy is a decision rule that describes the actions a player will take at each decision point.

Mixed Strategy is based on the following criteria.

- Maximin ≠ Minimax
- No Saddle Point
- Value of game exist
- Method: Probability, Dominance Property

Let us Sum Up

In this unit, the students have learned to understand the Pure and Mixed Strategy

Check your progress

1.Zero sum games are the one in which there are two agents whose actions must alternate and in which the utility values at the end of the game are always the same.

a. True

b. False

2.Zero sum game has to be a _______ game.

a. Single player

b. Two player

c. Multiplayer

d. Three player

3.A game can be formally defined as a kind of search problem with the following components.

a. Initial State

b. Successor Function

c. Terminal Test

d. All of the mentioned

Glossary

Pure Strategy: Particular course of action that are selected by player is called pure strategy (course of action). i.e each player knows in advance of all strategies out of which he always selects only one particular strategy regardless of the other players strategy

Mixed Strategy: Course of action that are to be selected on a particular occasion with some fixed probabilities are called mixed strategies. i.e. there is a probabilistic situation and objective of the players is to maximize expected gain or minimize expected losses by making choice among pure strategy with fixed probabilities.

Answers to check your progress

1.b

2.c

3.d

Suggested Readings

1. Hamdy A. Taha, Operations Research-An introduction, Pearson Education, 8th Edition / Prentice Hall of India, 2007.
2. Tulsian, P. C., Vishal Pandey, Quantitative Techniques – Theory and Problems, Pearson Publications,2006.
3. A. Ravindren, Don T. Phillips and James J. Solberg, Operations Research Principles and Practice, John Wiley and Sons, 2nd edition, 2000.

Unit -15
Maximin–Minimax Principle

STRUCTURE

Overview

In this Unit we are going to deal with the pure strategy and mixed strategy.

Objectives

After studying this unit, the students will be able:

- to understand and solve the Maximin– Minimax Principle

15.1. Maximin – Minimax Principle

Minimax means Minimum of Column Maxima. It is to minimize the maximum loss. (defensive). Maximin means Maximum of Row Minima. It is to maximize the minimum gain (offensive).

Strategy is based on the Maximin–Minimax Principle

- Pure: Maximin = Minimax
- Mixed: Maximin ≠ Minimax

For every finite two-person zero-sum game,

- Consider a game with two players A and B in which player A has m strategies (moves) and player B has n strategies (moves).
- The game can be described in the form of a payoff matrix such that the cell entry aij is the payment to A in A's payoff matrix when A chooses the i^{th} strategy and B chooses the j^{th} strategy.
- There is a number *v*, called the value of the game

- There is a mixed strategy for Player A such that A's average gain is at least v no matter what Player B does, and
- There is a mixed strategy for Player B such that B's average loss is at most v no matter what Player A does.
- Player A wishes to maximize his gain while player B wishes to minimize his loss.
- Since player A would like to maximize his minimum gain, we obtain the maximin value for player A and the corresponding strategy is called the maximin strategy. Player A's corresponding gain is called the maximin value or lower value (*v*) of the game.
- At the same time, player B wishes to minimize his maximum loss, we obtain the minimax value for player B and the corresponding strategy is called the minimax strategy. Player B's corresponding loss is called the minimax value or upper value(*v*) of the game.
- Usually the minimax value is greater than or equal to the maximin value. If the equality holds, i.e., then the corresponding pure strategies are called optimal strategies and the game is said to have a saddle point.

$$\max_i \min_j a_{ij} = \min_j \max_i a_{ij} = v$$

15.2. Solved Problems

Solve the following game whose payoff matrix is given below by Maximin–Minimax Principle. For the game with the following payoff matrix, determine the best strategies for Player A and B, and also determine the value of the game.

	B1	B2	B3	B4	B5
A1	2	4	3	8	4
A2	5	6	3	7	8

A3	**6**	**7**	**9**	**8**	**7**
A4	**4**	**2**	**8**	**4**	**2**

	B_1	B_2	B_3	B_4	B_5	**Row Minima**
A_1	2	4	3	8	4	2
A_2	5	6	3	7	8	3
A_3	6	7	9	8	7	6
A_4	4	2	8	4	2	2
Column Maxima	6	7	9	8	8	

Maximin–Minimax Principle

Maximin (Maximum of Row Minima); Minimax (Minimum of Column Maxima) Maximin = Minimax

6 = 6

Pure Strategy Saddle Point = A3B1

Value of the game = 6

Let us Sum Up

In this unit, the students have learned to understand and solve the Maximin–Minimax Principle

Check your progress

1.The initial state and the legal moves for each side define the ___________ for the game.

a. Search Tree

b. Game Tree

c. State Space Search

d. Forest

2.The minimax algorithm computes the minimax decision from the current state. It uses a simple recursive computation of the minimax values of each successor state, directly implementing the defining equations. The recursion proceeds all the way down to the leaves of the tree, and then the minimax values are backed up through the tree as the recursion unwinds.

a. True
b. False

3.What is the saddle point?

a. Point where function has maximum value

b. Point where function has minimum value

c. Point where function has zero value

d. Point where function neither have maximum value nor minimum value

Glossary

Minimax: Minimum of Column Maxima. It is to minimize the maximum loss. defensive).

Maximin: Maximum of Row Minima. It is to maximize the minimum gain (offensive).

Answers to check your progress

1.b

2.a

3.d

Suggested Readings

1. Hamdy A. Taha, Operations Research-An introduction, Pearson Education, 8th Edition / Prentice Hall of India, 2007.

2. Tulsian, P. C., Vishal Pandey, Quantitative Techniques – Theory and Problems, Pearson Publications,2006.

3. A. Ravindren, Don T. Phillips and James J. Solberg, Operations Research Principles and Practice, John Wiley and Sons, 2nd edition, 2000.

Unit-16
Dominance Property

STRUCTURE

Overview

If one strategy of a player dominates over the other strategy in all conditions then the later strategy can be ignored.

Objectives

After studying this unit, the students will be able to understand and solve the Dominance Property

16.1. Dominance Property

- Dominant property or strategy states that if one strategy of a player dominates over the other strategy in all conditions then the later strategy can be ignored.
- The rule of dominance are used to reduce the size of payoff matrix. These rule helps in deleting certain rows and columns of the payoff matrix that are inferior (less attractive).

Dominance Principle

- **Row (Minima):** If all the elements of a row (say ith row) are less than or equal to the corresponding elements of any other row (say jth row), then the ith row is dominated by the jth row and can be deleted from the matrix.

Column (Maxima): If all the elements of a column (say ith column) are greater than or equal to the corresponding elements of any other column (say jth column), then the ith column is dominated by the jth column and can be deleted from the matrix.

16.2. Solved Problems

Solve the following game whose payoff matrix is given below by Dominance Principle. For the game with the following payoff matrix, determine the best strategies for Player A and B, and also determine the value of the game.

	B1	B2	B3	B4	B5
A1	**2**	**4**	**3**	**8**	**4**
A2	**5**	**6**	**3**	**7**	**8**
A3	**6**	**7**	**9**	**8**	**7**
A4	**4**	**2**	**8**	**4**	**2**

	B_1	B_2	B_3	B_4	B_5
~~A_1~~	~~2~~	~~4~~	~~3~~	~~8~~	~~4~~
A_2	5	6	3	7	8
A_3	6	7	9	8	7
A_4	4	2	8	4	2

Row Minima – Minimum

A_1	$2\downarrow$	$4\downarrow$	$3 =$	8	$4\downarrow$
A_2	5	6	3	$7\downarrow$	8

A_1 & A_2 – No Dominance

A_1	$2\downarrow$	$4\downarrow$	$3\downarrow$	$8 =$	$4\downarrow$
A_3	6	7	9	8	7

A_1 – Dominance

	B_1	B_2	B_3	B_4	B_5
A_2	5	6	3	7	8
A_3	6	7	9	8	7
~~A_4~~	~~4~~	~~2~~	~~8~~	~~4~~	~~2~~

Row Minima – Minimum

A_2	5↓	6↓	3↓	7↓	8
A_3	6	7	9	8	7↓

A_2 & A_3 – No Dominance

A_2	5	6	3↓	7	8
A_4	4↓	2↓	8	4↓	2↓

A_2 & A_3 – No Dominance

A_3	6	7	9	8	7
A_4	4↓	2↓	8↓	4↓	2↓

A_4 – Dominance

	B_1	B_2	B_3	B_4	B_5
A_2	5	~~6~~	3	7	8
A_3	6	~~7~~	9	8	7

Column Maxima – Maximum

B_1	B_2
5	6↑
6	7↑

B_2 – Dominance

	B_1	B_3	B_4	B_5
A_2	5	3	~~7~~	8
A_3	6	9	~~8~~	7

Column Maxima – Maximum

B_1	B_2
5	6
6	7

B_2 – Dominance

B_1	B_3
5	3
6	9

No Dominance

B_1	B_4
5	7
6	8

B_4 – Dominance

	B_1	B_3	B_5
A_2	5	3	~~8~~
A_3	6	9	~~7~~

Column Maxima – Maximum

B_1	B_2
5	6
6	7

B_2 – Dominance

B_1	B_3
5	3
6	9

No Dominance

B_1	B_4
5	7
6	8

B_4 – Dominance

B_1	B_5
5	8
6	7

B_5 – Dominance

$$\begin{array}{ccc} & B_1 & B_3 \\ A_2 & \cancel{5} & \cancel{3} \\ A_3 & 6 & 9 \end{array}$$

Row Minima – Minimum

$$\begin{array}{ccc} A_2 & 5 \downarrow & 3 \downarrow \\ A_3 & 6 & 9 \end{array}$$

A_2 – Dominance

Column Maxima – Maximum

$$\begin{array}{cc} B_1 & B_3 \\ 6 & 9 \uparrow \end{array}$$

B_1 – Dominance

Row Minima – Minimum

$$\begin{array}{ccc} A_2 & 5 \downarrow & 3 \downarrow \\ A_3 & 6 & 9 \end{array}$$

A_2 – Dominance

Column Maxima – Maximum

$$\begin{array}{cc} B_1 & B_3 \\ 6 & 9 \uparrow \end{array}$$

B_1 – Dominance

$$\begin{array}{cc} & B_1 \\ A_3 & 6 \end{array}$$

Pure Strategy

Saddle Point = A_3B_1

Value of the game = 6

Let us Sum Up

In this unit, the students have learned to understand and solve the Dominance Property

Check your progress

1. A mixed strategy game can be solved by _______________.
   a. Simplex method
   b. Hungarian method
   c. Graphical method
   d. Degeneracy
2. When the sum of gains of one player is equal to the sum of losses to another player in a game, this situation is known as _______________.
   a. two-person game
   b. Two-person zero-sum game
   c. zero-sum game
   d. non-zero-sum game

3. A game is said to be strictly determinable if_______________.

   a. maximin value equal to minimax value

   b. maximin value is less than or equal to minimax value

   c. maximin value is greater than or equal to minimax value

   d. maximin value is not equal to minimax value

Glossary

Dominant property:	If one strategy of a player dominates over the other strategy in all conditions then the later strategy can be ignored.
The rule of dominance:	It is to reduce the size of payoff matrix. These rule helps in deleting certain rows and columns of the payoff matrix that are inferior (less attractive).

Answers to check your progress

1.c

2.a

3.a

Suggested Readings

1. A. Ravindren, Don T. Phillips and James J. Solberg, Operations Research Principles and Practice, John Wiley and Sons, 2nd edition, 2000.
2. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.

Block-5: Introduction

Block-5: Queuing and Simulation has been divided in to four Units. **Unit-17: Queuing System** deals with main aspects Queuing System and its components.

Unit-18: Queuing models: Birth and Death Model explains about the important aspects of Queuing models: Birth and Death Model and Solved Problems.

Unit-19: Simulation deals with main aspects Simulation and its components.

Unit-20: Simulation Models describes about Simulation Models and Solved Problems.

In all the units of Block -5 **Queuing and Simulation**, the Check your progress, Glossary, Answers to check your progress and Suggested Reading has been provided and the Learners are expected to attempt all the Check your progress as part of study.

Unit-17

Queuing System

STRUCTURE

Overview

In this chapter we are going to deal about the queuing system. It is a theory of mathematics of waiting lines.

Objectives

After studying this unit, the students will be able:

- to understand the queuing system.

17.1. Queuing System

- Queuing system or theory is the mathematics of waiting lines.
- It is extremely useful in predicting and evaluating system performance.
- It is to find the best level of service
- Analytical modeling (using formulas) can be used for many queues Characteristics of Queuing Systems
- Customer: refers to anything that arrives at a facility and requires service, e.g., people, machines, trucks, emails, packets, frames.
- Server: refers to any resource that provides the requested service, e.g., repairpersons, machines, runways at airport, host, switch, router, disk drive, algorithm

System	Customers	Server
Reception desk	People	Receptionist
Hospital	Patients	Nurses
Airport	Airplanes	Runway
Production line	Cases	Case-packer
Road network	Cars	Traffic light
Grocery	Shoppers	Checkout station
Computer	Jobs	CPU, disk, CD
Network	Packets	Router

The Queuing system or theory is described by four elements:

- Input
- Service Mechanism
- Queue Discipline
- Customer's behavior Input
- Size: Infinite or limited
- Arrival Pattern (Poisson Distribution): fixed or random; Measured by Mean arrival rate (λ)
- Service Mechanism
- Size: Number of servers (or channels)
- Service Pattern (Negative Exponential Distribution): fixed or random; Measured by Mean service rate (μ)
- Queue Discipline
- First In First Out (FIFO) – Traffic, ATM etc.
- Last In First Out (LIFO) – Emergency: Patient, Ambulance; Customer who comes for first class ticket etc.
- Service in Random Order (SIRO) – Online Ticket booking Customer's behavior
- Balking – A customer may leave the queue because the queue is too long and he has no time to wait, or there isn't sufficient waiting space.

- Reneging – This occurs when a waiting customer leaves the queue due to impatience. He joins the queue (thinking it will move quickly), but gets impatient after some time and then decides to leave.
- Priorities – In certain applications, some customers are served before others regardless of their order of arrival. These customers have priority over others.
- Jockeying – Customers may jockey from one line to another, i.e. move from one line to another.

Kendall's Notation – A / B / S

A = Arrival rate distribution

(M for Poisson, D for deterministic, and G for general)

B = Service rate distribution

(M for exponential, D for deterministic, and G for general)

c = Number of servers or channels

Queuing Model (Kendall Notation)	**Example**
Simple system (M / M / 1)	Customer service desk in a store
Multiple server (M / M / s)	Airline ticket counter
Constant service (M / D / 1)	Automated car wash
General service (M / G / 1)	Auto repair shop
Limited population (M / M / s / ∞ / N)	An operation with only 12 machines that might break

Applications of queuing system

- Customers: Counter like
  - Sales
  - Supermarket
  - Petrol Pump
  - Doctor

    - ATM
    - Ticketing
    - Bank etc.
- Trains: Railway station
- Planes: Airport
- Goods: Manufacturing
- Vehicles: Traffic System

Let us Sum Up

In this unit, the students have learned to understand the queuing system.

Check your progress

1. The total opportunity cost matrix is obtained by doing __________.
    a. row operation on row opportunity cost matrix
    b. column operation on row opportunity cost matrix
    c. column operation on column opportunity cost matrix
    d. none of the above
2. A game is said to be fair if ______________.
    a. lower and upper values are zero
    b. only lower value to be zero
    c. only upper value to be zero
    d. lower and upper values are not equal to zero
3. In game theory, the outcome or consequence of a strategy is referred to as the
    a. payoff.
    b. penalty.
    c. reward.
    d. end-game strategy

Glossary

Queuing system or theory: The mathematics of waiting lines in predicting and evaluating system performance to find the best level of

	service. Analytical modeling (using formulas) can be used for many queues
Customer:	Anything that arrives at a facility and requires service, e.g., people, machines, trucks, emails, packets, frames.
Server:	Any resource that provides the requested service, e.g., repairpersons, machines, runways at airport, host, switch, router, disk drive, algorithm

Answers to check your progress

1. b
2. a
3. d

Suggested Readings

1. A. Ravindren, Don T. Phillips and James J. Solberg, Operations Research Principles and Practice, John Wiley and Sons, 2nd edition, 2000.
2. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, “Introduction to Operation Research”, McGraw Hill, 11th edition, ISSN: 9354601200, 2021.

Unit -18

Queuing models: Birth and Death Model

STRUCTURE

Overview

This gives an overall view of the birth and death model of queuing.

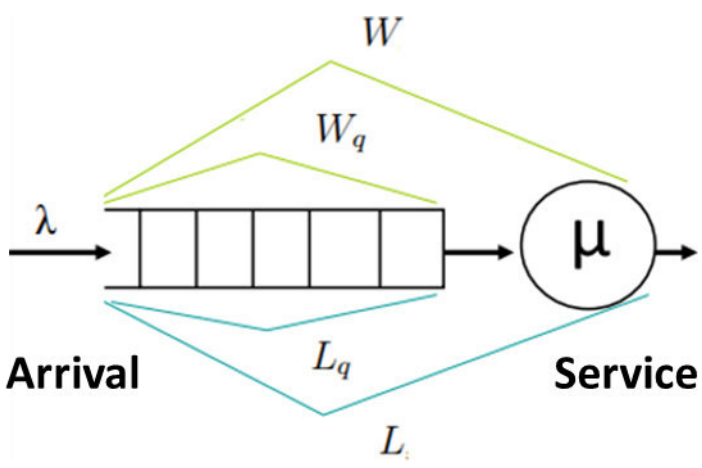
Objectives

After studying this unit, the students will be able

- to understand and solve the queuing models: Birth and Death Model

18.1. Queuing models: Birth and Death Model

In queueing theory the birth–death process is the most fundamental example of a queueing model, the M/M/C/K (in complete Kendall's notation) queue. This is a queue with Poisson arrivals, drawn from an infinite population, and C servers with exponentially distributed service times with K places in the queue.



Single Server Queuing System (M/M/1)

- Poisson arrivals
- Arrival population is unlimited
- Exponential service times
- All arrivals wait to be served
- λ is constant
- μ > λ (average service rate > average arrival rate)

Measuring Queue Performance – ρ; Lq; L; Wq; W ρ = utilization factor

$$\rho = \frac{\lambda}{\mu}$$

(probability of all servers being busy)

18.2. Solved Problems

Customers arrive at a sales counter manned by a single person according to a poison process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 30 per hour. Determine

- **Average queue length of a customer.**
- **Average waiting time of a customer.**

Given:

- Mean Arrival rate (λ) = 20 per hour
- Mean Service rate (μ) = 30 per hour Calculate:
- Average queue length of a customer (Lq)
- Average waiting time of a customer (Wq)

$$\rho = \frac{\lambda}{\mu}$$

ρ = Utilization factor

Average queue length of a customer (Lq)

$$L_q = \frac{\rho^2}{1 - \rho}$$

Average waiting time of a customer (Wq)

$$W_q = L_q/\lambda$$

Let us Sum Up

In this unit, the students have learned to understand and solve the queuing models: Birth and Death Model

Check your progress

1. Games without saddle point require players to play
   a. mixed strategies
   b. pure strategies
   c. both (a) and (b)
   d. none of these
2. To find the optimal solution, we apply ______________
   a. LPP
   b. VAM
   c. MODI Method
   d. Rim
3. For maximization in TP , the objective is to maximize the total ______
   a. Solution
   b. Profit Matrix
   c. Profit
   d. None of the above

Glossary

Birth–death queueing model: The most fundamental example of a queueing model, the M/M/C/K (in complete Kendall's notation) queue. This is a queue with Poisson arrivals, drawn from an infinite population, and C servers with exponentially distributed service times with K places in the queue.

Answers to check your progress

1.c

2.c

3.c

Suggested Readings

1. A. Ravindren, Don T. Phillips and James J. Solberg, Operations Research Principles and Practice, John Wiley and Sons, 2nd edition, 2000.
2. Frederick Hillier, Gerald J. Lieberman, Bodhibroto Nag, Preetam Basu, "Introduction to Operation Research", McGraw Hill, 11th edition, ISSN: 9354601200, 2021.

Unit-19

Simulation

STRUCTURE

Overview

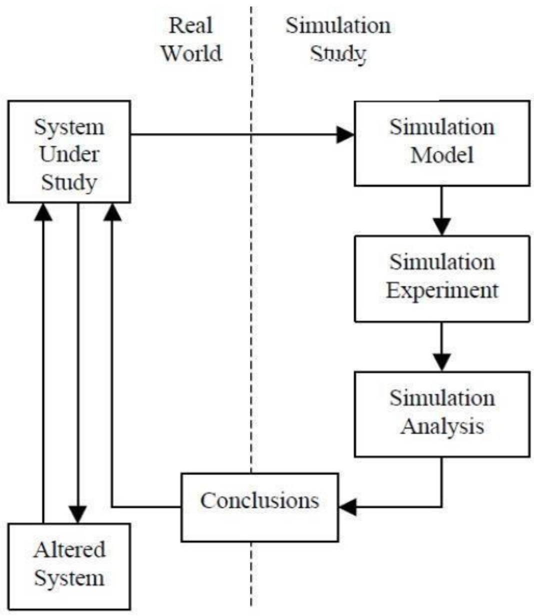
It is a process of simulation of a real life situation.

Objectives

After studying this unit, the students will be able:

- to understand the simulation.

19.1. Simulation



- Simulation is the imitation of the operation of a real-world process or system over time.

- Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose of either understanding the behavior of the system and/or evaluating various strategies for the operation of the system.

The types of Simulation

Based on State variable is Known or Uncertainty

1. Deterministic (Known)
2. Stochastic (Uncertainty – Probabilistic) Based on time evolution in the model
3. Static (Time evolution is not important)
4. Dynamic (Time evolution is important) Based on changes in the state of the system
5. Continuous System (changes in the state of the system occur only at discrete points in time)
6. Discrete System (changes in the state of the system occur only at discrete points in time)

Advantages of simulation

- Simulation is flexible and straightforward technique.
- It is useful in solving problems where all values of the variables are either not known or partially known.
- It can be used to analyze large and complex real world system that cannot be solved by conventional quantitative techniques models.
- In situations where it is difficult to predict or identify bottlenecks, Simulation is used to foresee these unknown difficulties.
- The simulation approach is useful to study a problem that involves uncertainty.
- It can be used to study existing systems without disrupting the ongoing operations.
- Proposed systems can be “tested” before committing resources.
- Simulation allows us to identify bottlenecks.

- Simulation allows us to gain insight into which variables are most important to system performance.

Applications of simulation

- Manufacturing facility
  - Designing and analyzing manufacturing systems
  - Evaluating a new military weapons system or tactics
  - Determining ordering policies for an inventory system
- Bank or other personal-service operation
  - Analyzing financial or economic systems
- Transportation/logistics/distribution operation
  - Designing and operating transportation facilities such as freeways, airports, subways, or ports
- Hospital facilities (emergency room, operating room, admissions), Fast-food restaurant, Supermarket
  - Evaluating designs for service organizations such as hospitals, post offices, or fast-food restaurants
- Computer network
  - Designing communications systems and message protocols for them
- Chemical plant

Let us Sum Up

In this unit, the students have learned to understand the simulation

Check your progress

1. When managers find standard queuing formulas inadequate or the mathematics unsolvable, they often resort to ____________ to obtain their solutions
   a. One
   b. VAM
   c. Simulation
   d. None of the above

2. In the standard queuing model, we assume that the queue discipline is ____________
   a. First come, first served
   b. Last come, first served
   c. First come, last served
   d. None of the above
3. The service time in the M/M/1 queuing model is assumed to be ____________
   a. Negative exponentially distributed
   b. Positively distributed
   c. Normal distribution
   d. None of the above

Glossary

Simulation: The process of designing a model of a real system and conducting experiments with this model for the purpose of either understanding the behavior of the system and/or evaluating various strategies for the operation of the system.

Answers to check your progress

1. c

2. a

3. a

Suggested Readings

1. Gupta. P. K, Man Mohan, Kanti Swarup: "Operations Research", Sultan Chand, 2008.
2. Goddard L.S., "Mathematical techniques of Operational Research", Elsevier, 2014.
3. Sharma J. K., "Operations Research Theory & Applications", Macmillan India Limited, fifth edition.2013

Unit-20
Simulation Models

STRUCTURE

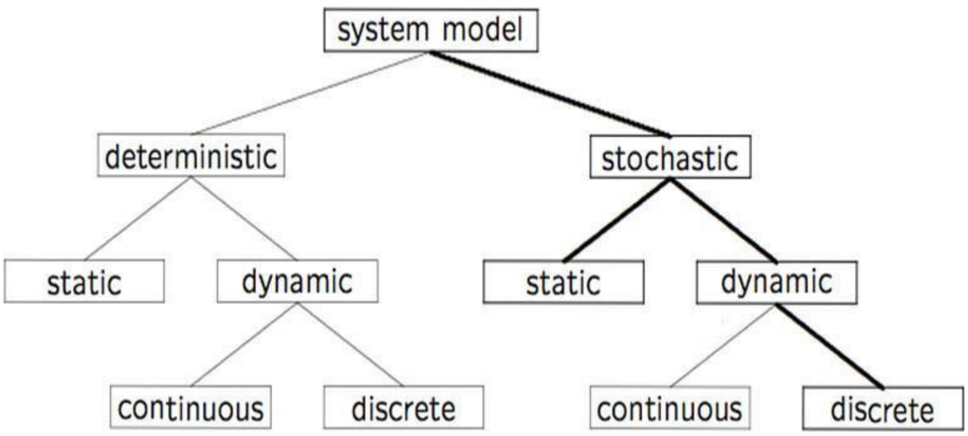
Overview

In this unit we are going to deal with probabilistic and discrete models.

Objectives

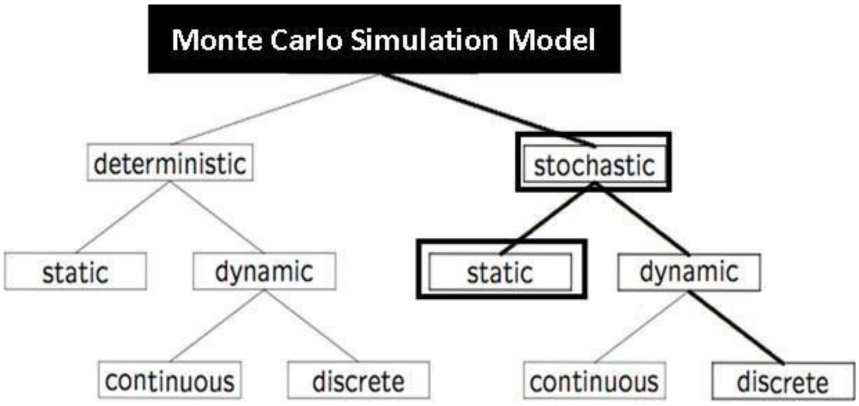
After studying this unit, the students will be able to understand and solve the simulation models.

20.1. Simulation Models



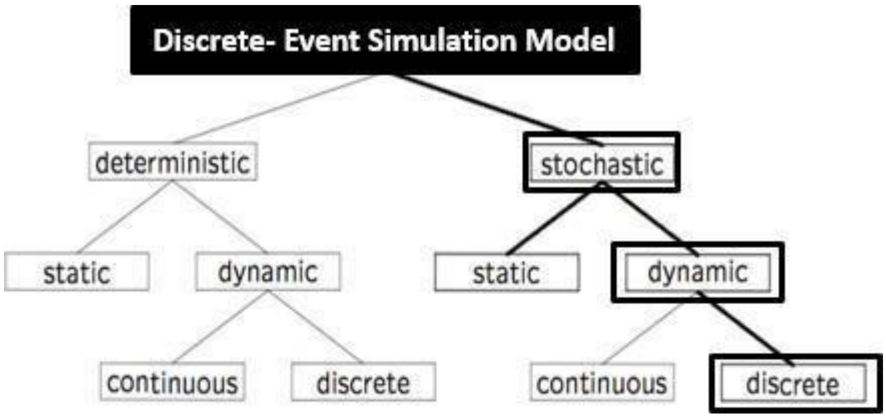
- Simulation model is a representation of a real system.
- Simulation is the process of designing a model of a real system and conducting experiments with this model.

Monte Carlo Simulation Model



- Monte Carlo is sampling method based upon probability distribution and the use of random numbers.
- Stochastic (Uncertainty – Probabilistic)
- Static (Time evolution is not important)

Discrete-Event Simulation Model



- A discrete-event simulation is one where changes in the state of the system occur instantaneously at random points in time as a result of the occurrence of discrete events.
- Stochastic (Uncertainty – Probabilistic)
- Dynamic (Time evolution is important)
- Discrete System (changes in the state of the system occur only at discrete points in time)

Solved Problems

The Lajwaab Bakery Shop keeps stock of a popular brand of cake. Previous experience indicates the daily demand as given below:

Daily demand	Probability
0	**0.01**
15	**0.15**
25	**0.20**
35	**0.50**
45	**0.12**
50	**0.02**

Consider the following sequence of random numbers: 21, 27, 47, 54, 60, 39, 43, 91, 25, 20. Using this sequence, simulate the demand for the next 10 days. Find out the stock situation, if the owner of the bakery shop decides to make 30 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data.

Using the daily demand distribution, we obtain a probability distribution as shown in the following table.

Table 1

Daily demand	Probability	Cumulative probability	Random Numbers
0	0.01	0.01	0
15	0.15	0.16	1-15
25	0.20	0.36	16-35
35	0.50	0.86	36-85
45	0.12	0.98	86-97
50	0.02	1.00	98-99

At the start of simulation, the first random number 21 generates a demand of 25 cakes as shown in table 2.

The demand is determined from the cumulative probability values in table

1. At the end of first day, the closing quantity is 5 (30-25) cakes. Similarly, we can calculate the next demand for others.

Table 2

Demand	Random Numbers	Next demand	Daily production = 30 cakes	
			Left out	Shortage
1	21	25	5	
2	27	25	10	
3	47	35	5	
4	54	35	0	
5	60	35		5
6	39	35		10
7	43	35		15
8	91	45		30
9	25	25		25
10	20	25		20
Total		**320**		**10**

Total demand = 320

Average demand = Total demand/no. of days

The daily average demand for the cakes = 320/10 = 32 cakes.

Let us Sum Up

In this unit, the students have learned to understand and solve the simulation models.

Check your progress

1. Simulation is a technique usually reserved for studying only the simplest and most straightforward of problems.

   a. True

   b. False

2. A simulation model is designed to arrive at a single specific numerical answer to a given problem.

   a. True

   b. False

3. Simulation typically requires a familiarity with statistics to evaluate the results.

   a. True

   b. False

Glossary

Monte Carlo simulation: Sampling method is based upon probability distribution and the use of random numbers.

Discrete-event simulation: One where changes in the state of the system occur instantaneously at random points in time as a result of the occurrence of discrete events.

Answers to check your progress

1. b

2.b

3.a

Suggested Readings

1. Gupta. P. K, Man Mohan, Kanti Swarup: “Operations Research”, Sultan Chand, 2008.

2. Goddard L.S., “Mathematical techniques of Operational Research”, Elsevier, 2014.

3. Sharma J. K., “Operations Research Theory & Applications”, Macmillan India Limited, fifth edition.2013

Annexure-Case studies

Case Study -1

Techway is an established company in the field of Robotics creating AI based solutions for its clients. The company has a state of the art R&D department in collaboration with a Japanese firm as part of its strategic alliance. In order to fulfill the mandatory CSR procedure of New Companies law 2013, Techway started a initiative in rural schools. The project was about the use of robots in class rooms for children with learning problems. The project was a huge success and lot of schools approached the company for its assistance in implementation of the project in their schools. As it was a CSR initiative, the company was not able to generate revenue through the project. The project manager approaches the GM of the company to discuss the issue. The project manager presents a clear view about how the project was started from CSR perspective and how it is consuming their resources now. The project manager suggests to complete the initiative while the GM wants to continue it as it has considerably increased the goodwill of the company.

Questions

1. Is it advisable to stop the initiative now?

2. How feasible it is to commercialize this venture?

Case Study - 2

Susrutha Meditech is a pharma company with a considerable market share in Asia Pacific. BiotechIndia is a research lab agency which works on new drug development. Susrutha approaches Biotech India to develop an indigenous drug for curing skin cancer based on Indian herbs. Biotech would just act as a consultant on payment basis while the rest would be taken care by Susrutha. The team member, Mr.Jose of this cancer drug had come up with a break through innovation during the project which would help in preventing the growth of all types of cancer cells. M.Jose knows very well if Sustrutha Company comes to know of this, they would get the entire patent and would not allow Biotech to take credit nor generate revenue.

Questions

1. What action would you suggest for Biotech India now?

2. As susrutha Meditech is the project sponsor, should the innovation be made known to them?

Case Study - 3

Info Global is an ITES company based in Delhi. The company had bagged a project from Government of India related to cyber security. Varun is a senior java Architect who is included in the project team. The team leader is an experienced new hand from Singapore branch of Info Global. The team leader is new to the Indian environment, especially to the Government procedures. As GOI is the sponsor for this project, there are lot of delays and bottlenecks in the 145 project due to red tapism. Varun identifies that the new team lead is having a tough time with the sponsor. He is also aware of the intensity and value of the project.

Questions

1. As a senior programmer, should Varun escalate the problem to the management?

2. Is it right on Varun's part to lend a helping hand to the new team lead to sort out things?

3.Offer your suggestions for the timely completion of the project.

Case Study - 4

A project comprising of eight tasks (A to H) has the following characteristics:

Tasks Preceding Tasks Time duration in weeks

Optimistic Most Likely Pessimistic

A -2, 4 ,12

B -10, 12 ,26

C- A, 8 ,9 ,10

D- A, 10 ,15, 20

E- A, 7, 7.5 ,15

F- B,C, 9, 9, 9

G- D, 3, 3.5, 7

H- E,F,G, 5, 5, 5

Questions

1.Draw the network diagram and carry out all related calculations.

2. Determine the critical path and mark it in the network.

3.What is the total project duration?

Case Study - 5

Read well is a bookstore which sells all genres of books. It the advent of e-commerce , the company wants to open its online book store and also develop an app for purchase. This project is given to the company doit.com.

Question

1. As the team leader of this project, prepare a project charter for the launching of the project.

Case Study - 6

Using the minimax criterion find the optimal strategies for the players in the following game.

	Player B				
Player A	8	10	13	16	9
	7	12	6	15	10
	9	18	9	13	25
	4	9	8	20	6

Case Study - 7

A machine costs Rs. 6000/-. The running cost and the resale value (salvage value) at the end of the year is given below.

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Operating maintenance cost	1200	1400	1600	1800	2000	2400	3000
Resale value	4000	2666	2000	1500	1000	600	600

Question

1.Find when the machine is to be replaced?

Case Study - 8

Solve the following assignment problem using Hungarian method. The matrix entries represent the processing times in hours.

		OPERATOR				
		1	**2**	**3**	**4**	**5**
JOB	**1**	9	11	14	11	7
	2	6	15	13	13	10
	3	12	13	6	8	8
	4	11	9	10	12	9
	5	7	12	14	10	14

Case Study - 9

Find the initial basic feasible solution of a transportation problem using Vogel's Approximation method

Destination					
	D1	**D2**	**D3**	**D4**	**Supply**
Source					
S1	190	300	500	100	**70**
S2	700	300	400	600	**90**
S3	400	100	600	200	**180**
Demand	**50**	**80**	**70**	**140**	**340**

Case Study -10

The automobile company manufactures around 100 cars. The production is more accurately described by the probability distribution given below.

Production per day	Probability
95	0.03
96	0.05
97	0.07
98	0.10
99	0.15
100	0.20
101	0.15
102	0.10
103	0.07
104	0.05
105	0.03

The finished cars are transported in a specially arranged lorry accommodating 101 cars.

(a) What will be the average number of cars waiting in the factory?

(b) What will be the average number of empty spaces in the lorry?

Using the following random numbers 68, 75, 29, 94, 08, 74, 36, 47, 31, 78, 38, 59, 91, 06, 31. Simulate the process.

Model End Semester Examination Question Paper

Master of Business Administration (MBA)

Course Code: **DCMBA-21**/ Course Title: **Quantitative Techniques**

Max. Marks: 70 **Time: 3 hours**

PART – A (10x2 =20 Marks)

Answer any **TEN** questions out of TWELVE questions

[All questions carry equal marks]

(1).Organize and list few applications of Operations research.
(2).List the characteristics of Operations Research.
(3).What are the different types of solving Linear Programming Problem (LPP)?
(4).Explain the difference between transportation and assignment problems.
(5).What is unbalanced AP? How to resolve it?
(6).Define Network analysis.
(7).Distinguish between PERT and CPM.
(8).What is Game theory?
(9).List out the methods for solving the game problem.
(10).What is saddle point?
(11).What are the uses of simulation?
(12).Write the characteristics of queuing theory.

PART – B (5X8=40 Marks)

Answer any **FIVE** questions out of SEVEN questions

[All questions carry equal marks]

(13).Solve the following LPP problem using graphical method

Maximise Z = 12X1 + 16X2

Subject to 10X1 + 20X2 ≤ 120

8X1 + 8X2 ≤ 80

X1, X2 ≥ 0

(14).Calculate basic feasible solution of a transportation problem using North West Corner rule.

Destination				
	D1	**D2**	**D3**	**Supply**
Source				
S1	2	7	4	**5**
S2	3	3	1	**8**
S3	5	4	7	**7**
S4	1	6	2	**14**
Demand	**7**	**9**	**18**	**34**

(15).Find the initial basic feasible solution of a transportation problem using Least cost method .

	Destination				
	D1	**D2**	**D3**	**D4**	**Supply**
Source					
S1	1	2	3	4	**6**
S2	4	3	2	0	**8**
S3	0	2	2	1	**10**
Demand	**4**	**6**	**8**	**6**	**24**

(16).A project scheduling has the following characteristics

Activity	Time in days
1 - 2	4
1 - 3	1
2 - 4	1
3 - 4	1
3 - 5	6
4 - 9	5
5 - 6	4
5 - 7	8
6 - 8	1
7 - 8	2
8 - 10	5
9 - 10	7

Prepare Network using CPM and determine critical path, duration and total float.

(17).Solve the following game

	Player B			
Player A	12	1	30	-10
	20	3	10	5
	-5	-2	25	0
	15	-4	10	6

(18).Discuss the solution of the following game

	Player B			
Player A	8	2	9	5
	6	5	7	18
	7	3	-4	10

(19).The automobile company manufactures around 150 scooters. The daily production varies from 146 to 154 depending upon the availability of raw materials and other working conditions.

Production per day	**Probability**
146	0.04
147	0.09
148	0.12
149	0.14
150	0.11
151	0.10
152	0.20
153	0.12
154	0.08

The finished scooters are transported in a specially arranged lorry accommodating 150 scooters. Using the following random numbers 80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57. Simulate the process to find out

(i).What will be the average number of scooters waiting in the factory?

(ii).What will be the average number of empty spaces in the lorry?

PART - C (1x10=10 Marks)

CASE STUDY (Covering the Whole Course)

(20).Assign the four tasks to four operators. Solve the following assignment problem using Hungarian method.

		OPERATORS			
		1	**2**	**3**	**4**
TASKS	**A**	20	28	19	13
	B	15	30	31	28
	C	40	21	20	17
	D	21	28	26	12

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